Section: Turing Machines - Building Blocks

1. Given Turing Machines $M_1$ and $M_2$

Notation for

- Run $M_1$
- Run $M_2$

$M_1 \rightarrow M_2$

$z$ represents any symbol in
2. Given Turing Machines M1 and M2

M1

\[ S \rightarrow H \]

M2

\[ S' \rightarrow H' \]

\[ M1 \xrightarrow{x} M2 \]

\[ S \rightarrow H \xrightarrow{x;R} z \xrightarrow{z;L} S' \rightarrow H' \]

\[ z \text{ represents any symbol in} \]
\[ x \text{ is an element of} \]
3. Given Turing Machines M1, M2, and M3

M1

\[ S \rightarrow H \]

M2

\[ S' \rightarrow H' \]

M3

\[ S'' \rightarrow H'' \]

x is an element of \( S \rightarrow H \rightarrow H' \)
y is any element except x from \( S' \rightarrow H' \)
z is any element from \( S'' \rightarrow H'' \)
More Notation for Simplifying Turing Machines

Suppose $\Gamma = \{a, b, c, B\}$

- $z$ is any symbol in $\Gamma$
- $x$ is a specific symbol from $\Gamma$

1. $s$ - start
2. $R$ - move right
3. $L$ - move left
4. $x$ - write $x$ (and don’t move)
5. $R_a$ - move right until you see an $a$
6. $L_a$ - move left until you see an $a$

7. $R_{\neg a}$ - move right until you see anything that is not an $a$

8. $L_{\neg a}$ - move left until you see anything that is not an $a$

9. $h$ - halt in a final state

10. $\overset{a,b}{\rightarrow} \overset{w}{\rightarrow}$

If the current symbol is $a$ or $b$, let $w$ represent the current symbol.
Example
Assume input string $w \in \Sigma^+$, $\Sigma = \{a, b\}$.
If $|w|$ is odd, then write a $b$ at the end of the string. The tape head should finish pointing at the leftmost symbol of $w$.

input: bab, output: babb
input: ba, output: ba

What is the running time?
Example

Assume input string $w \in \Sigma^+, \Sigma = \{a, b\}$, $|w| > 0$

For each $a$ in the string, append a $b$ to the end of the string.

input: $abbabb$, output: $abbabbbb$

The tape head should finish pointing at the leftmost symbol of $w$. 
Turing’s Thesis Any computation that can be carried out by a mechanical means can be performed by a TM.

Definition: An *algorithm* for a function $f: \mathbb{D} \rightarrow \mathbb{R}$ is a TM $M$, which given input $d \in \mathbb{D}$, halts with answer $f(d) \in \mathbb{R}$.

Example: $f(x + y) = x + y$, $x$ and $y$ unary numbers.

\[
\begin{align*}
\text{start with:} & \quad 111 + 1111 \\
\uparrow
\end{align*}
\]

\[
\begin{align*}
\text{end with:} & \quad 1111111 \\
\uparrow
\end{align*}
\]
Example: Copy a String, $f(w) = w0w$, $w \in \Sigma^*$, $\Sigma = \{a, b, c\}$

Denoted by $C$

\begin{align*}
\text{start with:} & \quad \text{abac} \\
\text{end with:} & \quad \text{abac}0\text{abac}
\end{align*}

Algorithm:

- Write a 0 at end of string
- For each symbol in string
  - make a copy of the symbol
Example: Shift the string that is to the left of the tape head to the right, denoted by $S_R$ (shift right)

Below, “ba” is to the left of the tape head, so shift “ba” to the right.

start with: aaBbабca

↑

end with: aaBBbaca

↑
Algorithm:

- remember symbol to the right and erase it
- for each symbol to the left do
  - shift the symbol one cell to the right
- replace first symbol erased
- move tape head to appropriate position
Example: Shift the string that is to the right of tape head to the left, denote by $S_L$ (shift left)

\[
\begin{align*}
\text{start with:} & \quad \text{babcaBba} \\
\uparrow \\
\text{end with:} & \quad \text{bacaBBba} \\
\uparrow
\end{align*}
\]

(similar to $S_R$)
Example: Add unary numbers
This time use shift.

Example: Multiply two unary numbers, \( f(x*y) = x*y \), \( x \) and \( y \) unary numbers. Assume \( x, y > 0 \).

\[
\begin{align*}
\text{start with:} & \quad 1111*11 \\
& \quad \uparrow \\
\text{end with:} & \quad 11111111 \\
& \quad \uparrow
\end{align*}
\]