Review

Regular Languages

- FA, RG, RE
- recognize

Context Free Languages

- PDA, CFG
- recognize

DFA:

Turing Machine:
**Turing Machine** (TM)

- invented by Alan M. Turing (1936)
- computational model to study algorithms

**Definition of TM**

- **Storage**
  - tape

- **actions**
  - write symbol
  - read symbol
  - move left (L) or right (R)

- **computation**
  - initial configuration
    * start state
    * tape head on leftmost tape square
    * input string followed by blanks
  - processing computation
    * move tape head left or right
    * read from and write to tape
  - computation halts
    * final state

**Formal Definition of TM**

A TM $M$ is defined by $M=(Q,\Sigma,\Gamma,\delta,q_0,B,F)$ where

- **Q** is finite set of states
- **\(\Sigma\)** is input alphabet
- **\(\Gamma\)** is tape alphabet
- **\(B\in\Gamma\)** is blank
- **\(q_0\)** is start state
- **\(F\)** is set of final states
- **\(\delta\)** is transition function

$\delta(q,a) = (p,b,R)$ means “if in state $q$ with the tape head pointing to an ’a’, then move into state $p$, write a ’b’ on the tape and move to the right”.
TM as Language recognizer

**Definition:** Configuration is denoted by $\vdash$.

If $\delta(q,a) = (p,b,R)$ then a move is denoted

\[
abaqabba \vdash ababpbba
\]

**Definition:** Let $M$ be a TM, $M=(Q,\Sigma,\Gamma,\delta,q_0,B,F)$. $L(M) = \{w \in \Sigma^* | q_0 w^* \vdash x_1 q_f x_2 \text{ for some } q_f \in F, x_1, x_2 \in \Gamma^*\}$

TM as language acceptor

$M$ is a TM, $w$ is in $\Sigma^*$,

- if $w \in L(M)$ then $M$ halts in final state
- if $w \notin L(M)$ then either
  - $M$ halts in non-final state
  - $M$ doesn’t halt

Example

$\Sigma = \{a, b\}$

Replace every second ‘a’ by a ‘b’ if string is even length.

- Algorithm
Example:

\[ L = \{ a^n b^n c^n | n \geq 1 \} \]

Is the following TM Correct?

**TM as a transducer**

TM can implement a function: \( f(w) = w' \)

- start with: \( w \)
  \[ \uparrow \]
- end with: \( w' \)
  \[ \uparrow \]
**Definition:** A function with domain $D$ is *Turing-computable* or *computable* if there exists TM $M=(Q,\Sigma,\Gamma,\delta,q_0,B,F)$ such that

$$q_0w \vdash^* q_f f(w)$$

$q_f \in F$, for all $w \in D$.

**Example:**

$f(x) = 2x$

$x$ is a unary number

Start with: 111

↑

End with: 111111

↑

Is the following TM correct?
Example:

$L = \{ww \mid w \in \Sigma^+\}, \Sigma = \{a, b\}$