1 Markov Chains

Equation 14.10 in the textbook describes the stationary distribution of a Markov chain. Show that the stationary distribution must be a left eigenvector of the transition matrix for the Markov chain and also indicate the eigenvalue. (The form of the transition matrix is described in the class notes.) Note that column vector $x$ is a left eigenvector of matrix $A$ if there exists a scalar eigenvalue $\lambda$ such that $x^TA = \lambda x$. This is a fairly trivial question that follows quite directly from the definitions of the terms, so you should have a very succinct answer.

2 HMMs

Rewrite the forward-backward algorithm using matrix-vector operations wherever possible. Use $.*$ (or some other clearly defined notation) to indicate the pointwise product, i.e, if $c = a.*b$, then $c[i] = a[i] * b[i]$.

3 HMM implementation

Implement the forward backward algorithm. (Your answer to the previous question should make this very easy, especially if you use a language like Matlab that has matrix-vector operations as primitives.) Verify your code on the student tracking example used in the class notes.

4 Viterbi Implementation

Implement the Viterbi algorithm and verify your code by finding the mostly likely path for the student with two consecutive meetings with no results. Also consider the following example: Two time steps of an HMM with 2 possible states 0,1 and no observations. The distribution at time step 0 is $P(s_0 = 1) = \frac{1}{3}$ and the state transition probabilities are: $P(s_1 = 1|s_0 = 1) = 1$, $P(s_1 = 1|s_0 = 0) = \frac{2}{5}$. Compare the Viterbi path with the highest probability states at each time step after running the forward backward algorithm on this HMM. You should see something strange - comment on this. (Note: You may be wonder how to use your code for an HMM with no observations. No observations is the same as all states having an identical, single observation that occurs with probability 1.0. What this means is that having completely useless observations is really the same as having no observations at all.)
5 Value of Information

Do problem 16.17.