Filtering Tricks and Extensions

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What we know so far

- Kalman filter
  - Assumes linear-Gaussian model
  - Exact, closed form solution given assumptions
- Particle filter
  - No modeling assumptions
  - Uses particles (simulation) to model distribution
  - May require many particles in some cases (= slow)

What if dynamics aren't linear?

- Kalman filter assumes:
  - $x_{k+1} = f(x_k) + \text{noise}$
  - $f$ assumed linear

- For non-linear $f$
  - $x_{k+1} = x_k + f'(x_k) \Delta u + \text{noise}$
  - First order linear approximation

Making it work

- Kalman filter linear update: $Ax$
- Extended kalman filter update: $A' \Delta x$
- $A'$ is the jacobian of the (non-linear) transition dynamics

Making it work II

- Must recompute Jacobian at every time step
- Derivatives taken at current mean
- $\Delta x$ must also be computed dynamically
- May also need to linearize observation model

EKF Pros and Cons

- Pros:
  - Works well for small step sizes
  - Computationally tractable in most cases
  - Distribution remains bounded size
- Cons:
  - Gets into trouble for large step sizes
  - Requires real time differentiation
**Unscented Filter**

- **Idea:**
  - Still represent distribution as a Gaussian
  - Use simulation to estimate parameters of Gaussian

- **Implementation:**
  - Pick a set of points with sample mean and sample covariance equal to current Gaussian (e.g., points evenly spaced at 1 SD contour)
  - Push these points through the particle filter equations
  - Compute a new sample mean and covariance for the weighted, post-simulation points

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**Surprising Facts about the UKF**

- It stinks less than the EKF 😅
- Mean and covariance at least as good as (actually better than) EKF - accurate up to second order
- No real time differentiation
- Fast and easy to implement

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**Why the UKF isn’t a silver bullet**

- Still assumes a Gaussian representation
- Can lead to (increasingly) bad approximations if distribution is not Gaussian
- …though probably no worse than EKF

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**Rao Blackwellised Particle Filter**

- **Idea:**
  - Sometimes it is possible to do parts of the filtering equations exactly, but not all
  - Sample the hard parts (particle filter)
  - Do the easy parts exactly (Kalman filter, or other method)

- **Example application**
  - Switching Kalman filter
  - Discrete variable selects from a set of possible linear
  - dynamics at each time step
  - Mixture Kalman filter
    - Sample discrete variables
    - Do exact KF computations for continuous variables

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**Mixture Kalman Filter Implementation**

- Distribution is a set of particles, where each particle is a Gaussian distribution
- Can combine with EKF, UKF, etc.
- Efficiency, accuracy depend upon
  - Number of particles
  - Size of sampled state space
  - Concentration of particles
  - Approximation quality (EKF, UKF)

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**Summary**

- **Kalman filter**
  - Linear/Gaussian model
  - Closed form solution
  - Exact when assumptions met

- **Extended Kalman Filter**
  - Nonlinearized Gaussian model
  - Closed form solution
  - Gaussian approximation to non-Gaussian reality

- **Unscented Filter**
  - Arbitrary model (almost)
  - Sample based approximation (for careful selection of sample points)
  - Fits Gaussian to samples

- **Particle Filter**
  - Arbitrary model (almost)
  - Sample based approximation
  - Quality depends upon size of particles, distribution

- **Rao-Blackwellized Particle Filter**
  - Arbitrary model (almost)
  - Combines sampling with closed form solution
  - Mixture of distributions approximates improves on particle filter accuracy