**Blocking Flows (Dimo)**

Idea: use multiple augmenting paths simultaneously.

Layered graph: by distance from source

Admissible arc: from layer $i-1$ to $i$

Admissible path: using only admissible arcs

Locating flow: saturates at least one arc on every admissible path

Lemma: Adding a blocking flow increases source-sink distance in residual network.

Proof: In new residual network, there is no $s-t$ path that only uses admissible arcs from previous residual network. So, every $s-t$ path in new residual network must be longer than shortest path in previous residual network since $(s, x)$ and $(x, t)$ distances are non-decreasing for all $x \in V$.

Corollary: $n$ blocking flows yields a maxflow

Q: What is the running time for finding a blocking flow?

ADVANCE: Take an admissible edge in forward direction

RETRACT: If no outgoing edge, go back along the edge leaving it
If you reach the sink, augment along path found and reduce capacity on edges by
the amount of flow augmentation.

Start from the source again.

Unit capacity graphs: Advance/Retreat
only once per edge \( \Theta(n) \) time for
blocking flow and \( \Theta(mn) \) overall.

Arbitrary capacities: Advance at most
\( m \) times per edge; nice only one edge
deleted per augmenting path \( \Theta(m^2) \)
time for blocking flow and \( \Theta(mn) \) overall.

Alternative bound for unit capacity graphs

After \( d \) iterations, residual maxflow is \( \leq \frac{1}{d} \)
since it has to be decomposable into paths of
length \( > d \).

\( \leq \) \# of iterations \( \leq d + \sqrt{m} \) for any \( d \)
Set \( d = \sqrt{m} \) to obtain running time
bound of \( O(m^{3/2}) \). This is better than
\( O(mn) \) since \( \sqrt{m} \leq n \).

For capacitated graphs,

- charge advances to augment/retract
- retreat deletes edge \( \Rightarrow \) \( O(m) \)
- augment deletes at least one
For each augment, at most \( O(n) \) advances charged to it

A augmenting path may have up to \( n \) edges

\[ \Rightarrow \text{Overall running time} = O(nm) \text{ per blocking flow} \Rightarrow O(mn^2) \text{ for flow algo} \]