DNF Counting

DNF: Disjunction (OR) of clauses/conjunction (AND)
Satisfiability of a DNF formula: Easy! Need to check if some clause can be satisfied.

Q: How many truth assignments satisfy a given DNF formula?
Why can’t we use the Monte Carlo method (counting via uniform sampling)?
Recall that # of samples $N \geq \frac{|U|}{|S|} \cdot \frac{1}{\varepsilon^2} \cdot \log\left(\frac{1}{\delta}\right)$

$|U| = 2^n$ for $n$ variables

$|S|$ can be as small as ONE!!

So, we would exponentially many samples.

Solution: Importance sampling (Karp, Luby, Madras)

Define $U$ as the multi-set of all clauses and $S$ as the circled ones. Can we now use the Monte Carlo method?

- Drawing a uniform random sample from $U$:
  - For each clause $C_i$, let $N_i$ be the # of assignments satisfying it.
  - Choose $C_i$ w.p. $\frac{N_i}{\sum N_i}$

A column chosen...
Choose a satisfying assignment for $C_i$ uniformly at random.

- Check if the sample is in $S$.
- Check the assignments in fixed order to determine if $C_i$ is the first clause satisfied by the sample.

$\# \text{ of samples} = \frac{|C_i|}{|S|} \leq \frac{1}{\epsilon^2} \log(1/\delta) \leq \frac{1}{\epsilon^2} \cdot \frac{1}{5^5} \cdot \log(1/\delta)$