Dual Fitting

- Analysis technique
- Construct feasible dual with dual objective $\geq \frac{1}{2}$ algorithmic objective

$\Rightarrow$ Approx. factor of $2$.

Analysis of greedy algorithm for set cover:

$$\min \sum c_s x_s \quad \mid \max \sum_{e \in E} y_e$$

$$\sum_{s \in S} x_s \geq 1$$
$s \subseteq S$

Dual: When you choose set $s$ in the greedy algorithm which covers $k$, new element, set $y_e = \frac{c_s}{k \ln n}$.

Lemma: The dual is feasible.

Proof: $\sum_{e \in E} y_e \leq \left( \frac{c_s}{c_{s-1}} + \frac{c_s}{c_{s-2}} + \cdots \right) \frac{1}{\ln n}$

$\leq c_s \left( \frac{1}{c_{s-1}} + \frac{1}{c_{s-2}} + \cdots + 1 \right) \frac{1}{\ln n}$

$\leq c_s \frac{\log n}{\log n} = c_s$.