Integrality gap = \max \text{ integer opt } \left\{ \text{ fractional opt } \right\} \ (\text{for minimization})

\text{Set cover LP}
\begin{align*}
\min \sum \xi s_i \\
\sum_{s \in S} x_s &\geq 1 \\
-x_s &\geq 0
\end{align*}

U = binary strings of length k

\mathcal{S} = \{ S_e : e \in U \}, \ \text{with } \xi_{e} = 1 \forall e \in S_e

\text{where } S_e = \{ e \in U : \# \text{ of } 1's \text{ in } e \text{ on } e = 0 \text{ and } \text{on } e = 1 \}

\text{Clearly } |\mathcal{S}| = 2^{k-1}

\exists x_e = \frac{1}{2^{k-1}}

Also, each element is in 2^{k-1} set \iff feasible and the cost 2

Lemma: Any integer solution must contain at least k sets.

Proof: If sets \( S_{e_1}, S_{e_2}, \ldots, S_{e_p} \) are chosen, then \( e_1, e_2, \ldots, e_p \)

must span the hyper cube. Otherwise, \( \exists e \in U \) s.t. \( e \) is orthogonal to each of \( e_1, e_2, \ldots, e_p \) and therefore \( e \notin S_{e_1}, S_{e_2}, \ldots, S_{e_p} \)

Thus, \( p \geq k \).