Due Date: November 10, 2008

Problem 1: [10pts] Let $D$ be a set of $n$ circular disks in $\mathbb{R}^2$. Show that the boundary of their union has $O(n)$ vertices. Describe a randomized algorithm to compute the union of $D$ whose expected running time is $O(n \log n)$.

Problem 2: [15pts] Let $S$ be a set of $n$ segments in $\mathbb{R}^2$, let $W$ be a vertical strip that contains all segments of $S$, and let $\chi(S)$ be the number of intersection points in $S$. Show that:

(i) If the endpoints of $S$ lie on the boundary of $W$, then $\chi(S)$ can be computed in $O(n \log n)$ time.

(ii) If $m$ of the segments in $S$ have their endpoints lying in the interior of $W$, then $\chi(S)$ can be computed in $O((m^2 + n) \log n)$ time. (Hint: Use duality.)

(iii) Use (i) and (ii) to show that $\chi(S)$ can be computed in $O(n^{3/2} \log^2 n)$ time.

Problem 3: [10pts] Let $S$ be a set of $n$ points in $\mathbb{R}^2$. Show that the number of triples in $S$ that form isosceles triangles is $O(n^{7/3})$.

Problem 4: [10pts] Describe an $O(n^3)$ time algorithm to compute a minimum-weight triangulation of a convex $n$-gon. (Hint: Use dynamic programming.)