**Due Date: October 25, 2011**

**Problem 1:** [10pts] Let $P = \langle p_0, \ldots, p_{n-1} \rangle$ and $Q = \langle q_0, \ldots, q_{n-1} \rangle$ be two nonintersecting convex polygons in $\mathbb{R}^2$. Show that the common tangent, both inner and outer, can be computed in $O(\log n)$ time. You can assume that the sequence of vertices of $P$ (and $Q$) is stored in an array.

**Problem 2:** [15pts] Let $P$ be a set of $n$ points in $\mathbb{R}^2$. We define a map $N : S^1 \to P$, where $N(u) = \text{arg max}_{p \in P} \langle p, u \rangle$. $N$ induces a subdivision $P^*$ of $S^1$. How fast can $P^*$ be computed? ($\langle p, u \rangle$ is the inner product of the two vectors.)

We call a pair of vertices $p, q \in P$ antipodal if there are two parallel lines $h_p, h_q$ passing through $p$ and $q$, respectively, so that $P$ lies between them. The width of $P$ is the minimum width of a strip that contains $P$. Show that a strip that realizes the width of $P$ contains an edge of $CH(P)$ and an antipodal pair on its boundary. Describe an $O(n \log n)$ algorithm for computing the width of $P$. (Hint: Use the function $N$.)

Describe an $O(n \log n)$ algorithm for computing the minimum-area rectangle (of arbitrary orientation) that contains $P$.

**Problem 3:** [10pts] Let $D$ be a set of $n$ circular disks in $\mathbb{R}^2$. Show that the union of disks in $D$ has $O(n)$ vertices and edges, and that it can be computed in $O(n \log n)$ randomized expected time.

**Problem 4:** [15pts] Let $S$ be a set of $n$ points in $\mathbb{R}^2$. For each point $p \in S$, define $\text{Vor}_f(p) = \{x \in \mathbb{R}^2 \mid \|xp\| \geq \|xq\| \forall q \in S\}$ and the farthest point Voronoi diagram $\text{Vor}_f(S) = \{\text{Vor}_f(p) \mid p \in S\}$. Show that $\text{Vor}_f(p)$ is nonempty and unbounded if and only if $p$ is a vertex of the convex hull of $S$, and that the edges of $\text{Vor}_f(S)$ form a tree.