Ray shooting from an edge. Given a $n$-vertex simple polygon $P$ and an edge $e$ of $P$, show how to construct a data structure to answer the following query in $O(\log n)$ time and $O(n)$ space: Given a ray $r$ whose origin lies on $e$ and which is directed into the interior or $P$, return the edge of $P$ that $r$ hits first.

Proof. (This is only a sketch of the proof. High level idea is that we will convert a query into a point location query in the dual space.)

For simplicity, assume that the fixed edge $e$ is oriented vertically. Given any point $p$, let $p^*$ denote the dual line of $p$. For any ray $r$ emanating from $e$, let $l(r)$ denote the line containing $r$, the dual of which is denoted by $l^*(r)$ (which is a point). The set of lines (rays) passing through edge $e = \langle p, q \rangle$ are dualized into the set of points in the slab between two horizontal lines $p^*$ and $q^*$.

Now, take an arbitrary edge $f \neq e$. Let $R(f)$ be the set of all rays which emanates from $e$ and which hits $f$ before it hits any other edge in $P$. Obviously, $l(r)$ is above a subset of points from $P$, call it $P_1$, and below the remaining set of points $P_2 = P - P_1$. In other word, point $l^*(r)$ lies above all dual lines from $P_1^*$ and below all dual lines from $P_2^*$. It is therefore easy to see that the dual of $R(f)$ is a convex region which lies within the slab between $p^*$ and $q^*$. The dual of $R(f)$’s for all edges from $P$ form a convex subdivision of the slab. Since there are $n$ convex regions (each corresponding to some edge from $P$), the overall complexity of the subdivision in the dual space is $O(n)$ (why?). To find out which edge in $P$ is hit first by a query ray $r$ emanating from $e$ is the same as locating the point $l^*(r)$ in this subdivision, which can be done in $O(\log n)$ time using a linear size data structure.