CPS 130: Homework 5

Amortized Analysis and Hashing
(CLRS: Chapters 17 and 11)

Date on which distributed: Thursday, October 11, 2001
Date on which due: Thursday, October 18, 2001

Note: Zero credit will be given for homeworks submitted late.

1. [Exc 17.1-3 CLRS]
A sequence of n operations is performed on a data structure. The ith operation costs i if i is an exact power of 2, and 1 otherwise. Use aggregate analysis to determine the amortized cost per operation. Show all working required to arrive at the solution.

(Aggregate analysis is described in Section 17.1 of CLRS.)

2. [Exc 17.3-5 CLRS]
Suppose that a counter begins at a number with b 1’s in its binary representation, rather than at 0. Show that the cost of performing n INCREMENT operations is $O(n)$ if $n = \Omega(b)$. (Do not assume that $b$ is a constant.)

3. [Exc 11.2-2 CLRS]
Demonstrate the insertion of the keys 5, 28, 19, 15, 20, 33, 12, 17, 10 into a hash table with collisions resolved by chaining. Let the table have 9 slots, and let the hash function be $h(k) = k \mod 9$. (You are not required to show the state of the hash table and its chains after each insertion. Just describe the final state by means of a figure akin to Figure 11.3 on p. 225 of CLRS.)

4. [Exc 11.3-1 CLRS]
Suppose we wish to search a linked list of length n, where each element contains a key $k$ along with a hash value $h(k)$. Each key is a long character string. How might we take advantage of the hash values when searching the list for an element with a given key?

For what kind of hash function $h$ would you expect your search method to work well? For what kind of $h$ might you expect otherwise? Justify your answers.