Sixth Homework Assignment

Write the solution to each problem on a single page. The deadline for handing in solutions is November 25.

Problem 1. (20 points). Let S be a set of n unit disks in the Euclidean plane, each given by its center and radius, which is one. Give an algorithm that decides whether any two of the disks in S intersect.

Problem 2. ($20 = 10 + 10$ points). Let S be a set of n points in the Euclidean plane. The *Gabriel graph* connects points $u, v \in S$ with a straight edge if

$$\|u - v\|^2 \quad \leq \quad \|u - p\|^2 + \|v - p\|^2$$

for every point p in S.

(a) Show that the Grabriel graph is a subgraph of the edge skeleton of the Delaunay triangulation.

(b) Is the Gabriel graph necessarily connected? Justify your answer.

Problem 3. ($20 = 10 + 10$ points). Consider a set of $n \geq 3$ closed disks in the Euclidean plane. The disks are allowed to touch but no two of them have an interior point in common.

(a) Show that the number of touching pairs of disks is at most $3n - 6$.

(b) Give a construction that achieves the upper bound in (a) for any $n \geq 3$.

Problem 4. ($20 = 10 + 10$ points). Let K be a triangulation of a set of $n \geq 3$ points in the plane. Let L be a line that avoids all the points.

(a) Prove that L intersects at most $2n - 4$ of the edges in K.

(b) Give a construction for which L achieves the upper bound in (a) for any $n \geq 3$.

Problem 5. (20 points). Let S be a set of n points in the Euclidean plane, consider its Delaunay triangulation and the corresponding filtration of alpha complexes,

$$S = A_1 \subset A_2 \subset \ldots \subset A_k.$$

Under what conditions is it true that A_i and A_{i+1} differ by a single simplex for every $1 \leq i \leq m - 1$?