

Seventh Homework Assignment

The purpose of this assignment is to help you prepare for the final exam. Solutions will neither be graded nor even collected.

Problem 1. (20 = 5 + 15 points). Consider the class of satisfiable boolean formulas in conjunctive normal form in which each clause contains two literals, $2\text{-SAT} = \{\varphi \in \text{SAT} \mid \varphi \text{ is } 2\text{-CNF}\}$.

- (a) Is $2\text{-SAT} \in \text{NP}$?
- (b) Is there a polynomial-time algorithm for deciding whether or not a boolean formula in 2-CNF is satisfiable? If your answer is yes, then describe and analyze your algorithm. If your answer is no, then show that $2\text{-SAT} \in \text{NPC}$.

Problem 2. (20 points). Let A be a finite set and f a function that maps every $a \in A$ to a positive integer $f(a)$. The PARTITION problem asks whether or not there is a subset $B \subseteq A$ such that

$$\sum_{b \in B} f(b) = \sum_{a \in A-B} f(a).$$

We have learned that the PARTITION problem is NP-complete. Given positive integers j and k , the SUM OF SQUARES problem asks whether or not A can be partitioned into j disjoint subsets, $A = B_1 \dot{\cup} B_2 \dot{\cup} \dots \dot{\cup} B_j$, such that

$$\sum_{i=1}^j \left(\sum_{a \in B_i} f(a) \right)^2 \leq k.$$

Prove that the SUM OF SQUARES problem is NP-complete.

Problem 3. (20 = 10+10 points). Let G be an undirected graph. A path in G is *simple* if it contains each vertex at most once. Specifying two vertices u, v and a positive integer k , the LONGEST PATH problem asks whether or not there is a simple path connecting u and v whose length is k or longer.

- (a) Give a polynomial-time algorithm for the LONGEST PATH problem or show that it is NP-hard.
- (b) Revisit (a) under the assumption that G is directed and acyclic.

Problem 4. (20 = 10 + 10 points). Let $A \subseteq 2^V$ be an abstract simplicial complex over the finite set V and let k be a positive integer.

- (a) Is it NP-hard to decide whether A has k or more disjoint simplices?
- (b) Is it NP-hard to decide whether A has k or fewer simplices whose union is V ?

Problem 5. (20 points). Let $G = (V, E)$ be an undirected, bipartite graph and recall that there is a polynomial-time algorithm for constructing a maximum matching. We are interested in computing a minimum set of matchings such that every edge of the graph is a member of at least one of the selected matchings. Give a polynomial-time algorithm constructing an $O(\log n)$ approximation for this problem.