Problem 1

The given events are represented as follows:
- Dallas Cowboys win = D
- Tony Dorsett rushes for 100 yards = T

The following data is given in question:
- \( P(D) = 0.66 \)
- \( P(T) = 0.27 \)
- \( P(D|T) = 0.90 \)

We need to find \( P(DT) \)

\[
P(DT) = P(T) \cdot P(D|T) = 0.27 \cdot 0.90 = 0.243
\]

Problem 2

Without loss of generality, assume that 3 is the correct answer. You get 5 points for correct answer and none for wrong.

Define a random variable \( X = \) number of points earned

\[
X(1) = 0 \\
X(2) = 0 \\
X(3) = 5 \\
X(4) = 0
\]

\[
E(X) = \sum_{k=1}^{4} p(k) \cdot X(k)
\]

Since the student is selecting answers at random, \( p(k) = \frac{1}{4}, \forall k \in (1, 4) \)

Substituting \( p(k) \) in the above equation would give \( E(X) = \frac{5}{4} \)

Problem 3

Number of ways to arrange is

\[
\frac{12!}{5! \cdot 2! \cdot 2! \cdot 2!}
\]

Problem 4

Given \( x = 3 + \frac{4}{x} \)

Continued fraction representation would be
\[ 3 + \frac{4}{3 + \frac{4}{3 + \frac{4}{x}}} \]

\[ x = 3 + \frac{4}{x} \]
\[ x^2 = 3x + 4 \]
\[ x^2 - 3x - 4 = 0 \]
\[ x^2 - 3x - 4 = 0 \]
\[ (x - 4) \cdot (x + 1) = 0 \]
\[ x = 4 \text{ or } x = -1 \]
For a positive solution, \( x = 4 \)

**Problem 5**

Basis Step: For \( k = 1 \), \( 12^{2k} - 11^k = 12^2 - 11 = 133 \), which is divisible by 133. Hence the base case holds.

Inductive Step: Assume \( \forall i \in \{1, k\}, 12^{2i} - 11^i = 133 \cdot q \)

Let's multiply both sides of above equation by \((12^2 + 11)\)

\( (12^{2k} - 11^k) \cdot (12^2 + 11) = 133 \cdot q \cdot (12^2 + 11) \)
\( 12^{2k+2} + 12^{2k} \cdot 11 - 11^2 \cdot 12^2 = 133 \cdot q \cdot (12^2 + 11) \)
\( 12^{2k+2} - 11^{n+1} \cdot 12^{2k} + 11 - 11^2 \cdot 12^2 = 133 \cdot q \cdot (12^2 + 11) \)
\( 12^{2(k+1)} - 11^{n+1} \cdot 12^{2k} + 11 - 12^2 \cdot 11 \cdot (12^{2(k-1)} + 11^{k-1}) = 133 \cdot q \cdot (12^2 + 11) \)

Using strong induction, \( 12^{2^k} + 11^{k-1} = 133r \), where \( r \) is an integer. Substitute this in the above equation

\( 12^{2(k+1)} - 11^{n+1} - 12^2 \cdot 11 \cdot 133 \cdot r = 133 \cdot q \cdot (12^2 + 11) \)
\( 12^{2(k+1)} - 11^{n+1} = 133 \cdot q \cdot (12^2 + 11) + 12^2 \cdot 11 \cdot r \)

which is a multiple of 133. Hence, the case for \( k + 1 \) is true. By induction, the theorem is true.

**Problem 6**

\[
\begin{align*}
P(1994|\text{Yellow}) &= \frac{P(1994, \text{Yellow})}{P(\text{Yellow})} = \frac{P(1994) \cdot P(\text{Yellow}|1994)}{P(\text{Yellow})} \\
P(\text{Yellow}) &= P(1994) \cdot P(\text{Yellow}|1994) + P(1996) \cdot P(\text{Yellow}|1996) \\
P(\text{Yellow}) &= \frac{1}{2} \cdot \frac{1}{5} + \frac{1}{2} \cdot \frac{14}{100} = 0.17
\end{align*}
\]

Substituting the above equation in the first one, we get

\[
P(1994|\text{Yellow}) = \frac{P(1994) \cdot P(\text{Yellow}, 1994)}{P(\text{Yellow})} = \frac{1}{5} \cdot \frac{1}{0.17} = \frac{10}{17}
\]
Problem 7

The counting can be done by summing up the counts of following two mutually exclusive cases.

Case 1: Professor wears BreastPlate

Sword or club - 4+2=6 ways
Helmet or nothing - 3+1=4 (Including the case for three helmets and nothing)
BreastPlate - 2
Gauntlets or nothing - 3+1=4
Plate Leggings or nothing - 1+1=2
Total = 6 * 4 * 2 * 4 * 2 = 384

Case 2: Professor wears ChainCoat

Sword or club - 4+2=6 ways
Helmet or nothing - 3+1=4
ChainCoat - 3
Gauntlets or nothing - 3+1=4
Chain Leggings or nothing - 2+1=3
Total = 6 * 4 * 3 * 4 * 3 = 864

Case 3: Professor wears neither of BreastPlate and ChainCoat

Sword or club - 4+2=6 ways
Helmet or nothing - 3+1=4
Nothing - 1
Gauntlets or nothing - 3+1=4
Plate Leggings, Chain Leggings or nothing - 1+2+1=4
Total = 6 * 4 * 1 * 4 * 4 = 384

Aggregate count including all these cases would be 384 + 864 + 384 = 1632

Problem 8

Let us group the numbers into following $n$ bins

$$\{2, 3\} \{3, 4\} \ldots \{g, q + 1\} \ldots \{2n, 2n + 1\}$$

The proof is based on the concept that two successive integers are relatively prime to each other.

Since, we are selecting more than $n + 1$ numbers out of $n$ bins, straightforward application of pigeon-hole principle tells that atleast two numbers should come from the same bin. Those two numbers will be relatively prime to each other because, they are successive numbers.