CompSci 590.6

Understanding Data:

Theory and Applications

Lecture 6

Mining Association Rules

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Today's Paper(s)

Fast Algorithms for Mining Association Rules

Agrawal and Srikant

VLDB 1994

18,603 citations on Google Scholar

One of the most cited papers in CS

- Acknowledgement:

The following slides have been prepared using several presentations of this paper available on the internet (esp. by Ofer Pasternak and Brian Chase)

Mining Association Rules

- Retailers can collect and store massive amounts of sales data
  - transaction date and list of items
- Association rules:
  - e.g. 98% customers who purchase “tires” and “auto accessories” also get “automotive services” done
  - Customers who buy mustard and ketchup also buy burgers
  - Goal: find these rules from just transactional data (transaction id + list of items)

Applications

- Can be used for
  - marketing program and strategies
  - cross-marketing
  - catalog design
  - add-on sales
  - store layout
  - customer segmentation

Notations

- Items $I = \{i_1, i_2, \ldots, i_m\}$
- D : a set of transactions
- Each transaction $T \subseteq I$
  - has an identifier TID
- Association Rule
  - $X \rightarrow Y$
  - $X, Y \subset I$
  - $X \cap Y = \varnothing$

Confidence and Support

- Association rule X→Y
- Confidence c = |Tr. with X and Y|/|Tr. with |X|
  - c% of transactions in D that contain X also contain Y
- Support s = |Tr. with X and Y| / |all Tr.|
  - s% of transactions in D contain X and Y.

Support Example

TID	Cereal	Beer	Bread	Bananas	Milk
1	X		X		X
2	X		X	X	X
3		X			X
4	X			X	
5			X		X
6	X				X
7		X		X	
8			X		

- Support(Cereal)
  - 4/8 = .5
- Support(Cereal → Milk)
  - 3/8 = .375

Confidence Example

TID	Cereal	Beer	Bread	Bananas	Milk
1	X		X		X
2	X		X	X	X
3		X			X
4	X			X	
5			X		X
6	X				X
7		X		X	
8			X		

- Confidence(Cereal → Milk)
  - 3/4 = .75
- Confidence(Bananas → Bread)
  - 1/3 = .33333…

Problem Definition

- Input
  - a set of transactions D
  - min support (minsup)
  - min confidence (minconf)
- Goal
  - generate all association rules that have
    - support >= minsup and
    - confidence >= minconf

X → Y is not a Functional Dependency

For functional dependencies

- F.D. = two tuples with the same value of of X must have the same value of Y
  - X → Y => XZ → Y (concatenation)
  - X → Y, Y → Z => X → Z (transitivity)

For association rules

- X → A does not mean XY→A
  - May not have the minimum support
  - Assume one transaction {AX}
- X → A and A → Z do not mean X → Z
  - May not have the minimum confidence
  - Assume two transactions {XA}, {AZ}

Divide into two subproblems

1. Find all sets of items (itemsets) that have support above the minimum support
   - #transactions containing them >= threshold
   - these are called large itemsets
2. Use large item sets to find rules with at least minimum confidence
   - Naïve algorithm:
   - For every large itemset p,
   - find all non-empty subsets of p
   - for each such subset q, if support(p)/support(q) >= minconf
   - output q → (p - q)

- Paper focuses on subproblem 1
  - if support is low, confidence may not say much
  - subproblem 2 in full version

- Two main algorithms: Apriori and AprioriTID

Determining Large Itemsets

- Algorithms make multiple passes over the data (D) to determine which itemsets are large
- First pass:
  - Count support of individual items
  - Determine which are large
- Subsequent Passes:
  - Use itemsets from previous passes sets to determine new potential" large itemsets ("candidate" large itemsets sets)
  - Count support for candidates from data (D) and remove ones not above minsup to get "actual" large itemsets
- Repeat

Notations

k-itemset	An itemset having k items.	
L_k	Set of large k-itemsets (those with minimum support). Each member of this set has two fields: i) itemset and ii) support count.	ACTUAL
C_k	Set of candidate k-itemsets (potentially large itemsets). Each member of this set has two fields: i) itemset and ii) support count.	POTENTIAL Used in both Apriori and AprioriTID
$\overline{C}_k$	Set of candidate k-itemsets when the TIDs of the generating transactions are kept associated with the candidates.	Used in AprioriTID

Algorithm Apriori

L_1 = *{large 1-itemsets}* ← Count individual item occurrences

For $(k = 2;\ L_{k-1} \neq \phi;\ k++)$ do begin

  $C_k = \text{apriori-gen}(L_{k-1});$ ← Generate new k-itemsets candidates

  forall transactions $t \in D$ do begin

    $C_t = \text{subset}(C_k, t)$

    forall candidates $c \in C_t$ do

      $c.count++;$

    end

  end

  $L_k = \{ c \in C_k \mid c.count \geq minsup \}$ ← Take only those with support >= minsup

end

$Answer = \bigcup_k L_k;$

Count individual item occurrences

Generate new k-itemsets candidates

count = 0

Find the support of all the candidates

C_t = candidates contained in t

increment count

Take only those with support >= minsup

Apriori-Gen

- **Join step**

insert into C_k

select $p.item_1, p.item_2, p.item_{k-1}, q.item_{k-1}$

from $L_{k-1} p, L_{k-1} q$

where $p.item_1 = q.item_1 , \ldots, \quad p.item_{k-2} = q.item_{k-2} , \quad p.item_{k-1} < q.item_{k-1}$

p and q are two large

(k-1)-itemsets identical in all k-2 first items.

Join by adding the last item of q to p

- **Prune step**

forall *itemsets* $c \in C_k$ do

forall *(k-1)-subsets s of c* do

if $(s \notin L_{k-1})$ then

delete *c from* C_k

Check all the subsets, remove all candidate with some "small" subset

Apriori-Gen Example - 1

Step 1: Join (k = 4)

Assume numbers 1-5 correspond to individual items

L_3

- {1,2,3}
- {1,2,4}

  {1,3,4}
- {1,3,5}
- {2,3,4}

C_4

- {1,2,3,4}

Apriori-Gen Example - 2

Step 1: Join (k = 4)

Assume numbers 1-5 correspond to individual items

L_3

- {1,2,3}
- {1,2,4}
  {1,3,4}
- {1,3,5}
- {2,3,4}

C_4

- {1,2,3,4}
- {1,3,4,5}

Apriori-Gen Example - 3

Step 2: Prune (k = 4)

- Remove itemsets that can't have the required support because there is a subset in it which doesn't have the level of support i.e. not in the previous pass (k-1)

L_3

- {1,2,3}
- {1,2,4}
  {1,3,4}
- {1,3,5}
- {2,3,4}

C_4

- {1,2,3,4}
- ~~{1,3,4,5}~~

No {1,4,5} exists in L_3
Rules out {1, 3, 4, 5}

Comparisons with previous algos (AIS, STEM)

L_{k-1} to C_k

- Read each transaction t
- Find itemsets p in L_k that are in t
- Extend p with large items in t and occur later in lexicographic order

L_3

- {1,2,3}
- {1,2,4}
- {1,3,4}
- {1,3,5}
- {2,3,4}

C_4

- {1,2,3,4}
- {1,2,3,5}
- {1,2,4,5}
- {1,3,4,5}
- {2,3,4,5}

t = {1, 2, 3, 4, 5}

5 candidates compared to 2 in Apriori

Correctness of Apriori

insert into C_k

select $p.item_1, p.item_2, p.item_{k-1}, q.item_{k-1}$

from $L_{k-1} p, L_{k-1} q$

where $p.item_1 = q.item_1, \ldots, \ p.item_{k-2} = q.item_{k-2}, \ \ p.item_{k-1} < q.item_{k-1}$

Show that $C_k \supseteq L_k$

- Any subset of large itemset must also be large
- for each p in L_k, it has a subset q in L_{k-1}
- We are extending those subsets q in Join with another subset q' of p, which must also be large
  - equivalent to extending L_{k-1} with all items and removing those

    whose (k-1) subsets are not in L_{k-1}
- Prune is not deleting anything from L_k

forall *itemsets* $c \in C_k$ do

forall *(k-1)-subsets s of c* do

if $(s \notin L_{k-1})$ then

delete *c from* C_k

Variations of Apriori

- In the k-th pass
  - Not only update C_k
  - update candidates C'_{k+1}
  - $C'_{k+1} \supseteq C_{k+1}$ since it is generated from L_k
  - Can help when the cost of updating and keeping in memory C'_{k+1} - C_{k+1} additional candidates is less than scanning the database

Subset Function

- Candidate itemsets in C_k are stored in a hash-tree (like a B-tree)
  - interior node = hash table
  - leaf node = itemsets
  - recall that the itemsets are ordered
- To find all candidates contained in a transaction t
  - if we are at a leaf
    - find which itemsets are contained in t
    - add references to them in the answer set
  - if we are at an interior node
    - we have reached it by hashing an item i
    - hash on each item that comes after i in t
    - repear
  - if we are at the root, hash on every item in t
- For any itemset c in a transaction t
  - the first item must be in the root

L_1 = *{large 1-itemsets}*
For $(k = 2;\ L_{k-1} \neq \phi;\ k++)$ do begin
  C_k = apriori-gen(L_{k-1});
  forall transactions $t \in D$ do begin
    C_t = subset(C_k, t)
    forall candidates $c \in C_t$ do
      $c.count++$;
    end
  end
  $L_k = \{ c \in C_k | c.count \geq minsup \}$
end
$Answer = \bigcup_k L_k$;

Problem with Apriori

- Every pass goes over the entire dataset
- Database of transactions is massive
  - Can be millions of transactions added an hour
- Scanning database is expensive
  - In later passes transactions are likely NOT to contain large itemsets
  - Don't need to check those transactions

L_1 = {*large 1-itemsets*}
For ($k = 2$; $L_{k-1} \neq \phi$; $k{+}{+}$) do begin
    C_k = apriori-gen(L_{k-1});
    forall transactions $t \in D$ do begin
        C_t = subset(C_k, t)
        forall candidates $c \in C_t$ do
            $c.count{+}{+}$;
        end
    end
    $L_k = \{ c \in C_k | c.count \geq minsup \}$
end
$Answer = \bigcup_k L_k$;

AprioriTid

- Also uses Apriori-Gen
- But scans the database D only once.
- Builds a storage set C^*_K
  - "bar" in the paper instead of *
- Members are of the form < TID, $\{X_k\}$ >
  - each X_k is a potentially large k-itemset present in the transaction TID.
  - For k=1, C^*_1 is the database
    - items i as {i}
- If a transaction does not have a candidate k-itemset, C^*_K will not contain anything for that TID
- C^*_K may be smaller than #transactions, esp. for large values of k
- For smaller values of k, it may be large

See the examples in the following slides and then come back to the algorithm

Algorithm AprioriTid

$L_1 = \{large\ 1\text{-}itemsets\}$

$\hat{C}_1 = database\ D;$

For $(\ k = 2;\ L_{k-1} \neq \phi\ ;\ k++)$ do begin

$C_k = \text{apriori-gen}(L_{k-1});$

$\hat{C}_k = \phi\ ;$

forall $entries\ t \in \hat{C}_{k-1}$ do begin

$C_t = \{c \in C_k | (c - c[k] \in t.set-of-items$

$\wedge (c - c[k-1]) \in t.set-of-items\};$

forall candidates $c \in C_t$ do

$c.count++;$

if $(C_t \neq \varphi)\ then\ \hat{C}_k += < t.TID, C_t >;$

end

end

$L_k = \{\ c \in C_k | c.count \geq minsup\}$

end

$Answer = \bigcup_k L_k;$

Count item occurrences

The storage set is initialized with the database

Generate new k-itemsets candidates

Build a new storage set

Determine candidate itemsets which are containted in transaction TID

Find the support of all the candidates

Remove empty entries

Take only those with support over minsup

Min support = 2

AprioriTid Example

Database

TID	Items
100	1 3 4
200	2 3 5
300	1 2 3 5
400	2 5

$\overline{C}_1$

TID	Set-of-Itemsets
100	{ {1}, {3}, {4} }
200	{ {2}, {3}, {5} }
300	{ {1}, {2}, {3}, {5} }
400	{ {2}, {5} }

L_1

Itemset	Support
{1}	2
{2}	3
{3}	3
{5}	3

Min support = 2

AprioriTid Example

L_1

Itemset	Support
{1}	2
{2}	3
{3}	3
{5}	3

Apriori-gen →

C_2

Itemset	Support
{1 2}	
{1 3}	
{1 5}	
{2 3}	
{2 5}	
{3 5}	

Now we need to compute the supports of C_2
without looking at the database D
from C^*_1

Min support = 2

AprioriTid Example

C_2

Itemset	Support
{1 2}	1
{1 3}	
{1 5}	
{2 3}	
{2 5}	
{3 5}	

$\overline{C}_1$

TID	Set-of-Itemsets
100	{ {1}, {3}, {4} }
200	{ {2}, {3}, {5} }
300	{ {1}, {2}, {3}, {5} }
400	{ {2}, {5} }

forall entries $t \in \overline{C}_{k-1}$ **do begin**

  // determine candidate itemsets in C_k contained

  // in the transaction with identifier t.TID

  $C_t = \{c \in C_k \mid (c - c[k]) \in t.\text{set-of-itemsets} \wedge$

    $(c - c[k-1]) \in t.\text{set-of-itemsets}\};$

  forall candidates $c \in C_t$ **do**

    c.count++;

  if $(C_t \neq \emptyset)$ **then** $\overline{C}_k$ += $< t.\text{TID}, C_t >$;

end

300 has both {1} and {2}
Support = 1
also add <300, {1, 2}> to C^*_2

Min support = 2

AprioriTid Example

C_2

Itemset	Support
{1 2}	1
{1 3}	2
{1 5}	
{2 3}	
{2 5}	
{3 5}	

$\overline{C}_1$

TID	Set-of-Itemsets
100	{ {1}, {3}, {4} }
200	{ {2}, {3}, {5} }
300	{ {1}, {2}, {3}, {5} }
400	{ {2}, {5} }

Add <100, {1, 3}> to C^*_2

Add <300, {1, 3}> to C^*_2

Min support = 2

AprioriTid Example

C_2

Itemset	Support
{1 2}	1
{1 3}	2
{1 5}	1
{2 3}	2
{2 5}	3
{3 5}	2

$\overline{C}_1$

TID	Set-of-Itemsets
100	{ {1}, {3}, {4} }
200	{ {2}, {3}, {5} }
300	{ {1}, {2}, {3}, {5} }
400	{ {2}, {5} }

Add the rest

Min support = 2

AprioriTid Example

C_2

Itemset	Support
{1 2}	1
{1 3}	2
{1 5}	1
{2 3}	2
{2 5}	3
{3 5}	2

$\overline{C}_2$

TID	Set-of-Itemsets
100	{ {1 3} }
200	{ {2 3}, {2 5}, {3 5} }
300	{ {1 2}, {1 3}, {1 5}, {2 3}, {2 5}, {3 5} }
400	{ {2 5} }

How C^*_2 looks

Min support = 2

AprioriTid Example

C_2

Itemset	Support
{1 2}	1
{1 3}	2
{1 5}	1
{2 3}	2
{2 5}	3
{3 5}	2

L_2

Itemset	Support
{1 3}	2
{2 3}	2
{2 5}	3
{3 5}	2

The supports are in place
Can compute L_2 from C_2

Min support = 2

AprioriTid Example

L_2

Itemset	Support
{1 3}	2
{2 3}	2
{2 5}	3
{3 5}	2

Apriori-gen →

C_3

Itemset	Support
{2 3 5}	

Next step

Min support = 2

AprioriTid Example

C_3

Itemset	Support
{2 3 5}	

$\overline{C}_2$

TID	Set-of-Itemsets
100	{ {1 3} }
200	{ {2 3}, {2 5}, {3 5} }
300	{ {1 2}, {1 3}, {1 5}, {2 3}, {2 5}, {3 5} }
400	{ {2 5} }

Look for transactions containing {2, 3} and {2, 5}

Add <200, {2,3,5}> and <300, {2,3,5}> to C^*_3

forall entries $t \in \overline{C}_{k-1}$ **do begin**
  // determine candidate itemsets in C_k contained
  // in the transaction with identifier t.TID
  $C_t = \{c \in C_k \mid (c - c[k]) \in t.\text{set-of-itemsets} \wedge$
    $(c - c[k-1]) \in t.\text{set-of-itemsets}\};$
  forall candidates $c \in C_t$ **do**
    c.count++;
  if $(C_t \neq \emptyset)$ **then** $\overline{C}_k$ += $< t.\text{TID}, C_t >$;
end

Min support = 2

AprioriTid Example

C_3

Itemset	Support
{2 3 5}	2

$\overline{C}_3$

TID	Set-of-Itemsets
200	{ {2 3 5} }
300	{ {2 3 5} }

L_3

Itemset	Support
{2 3 5}	2

C^*_3 has only two transactions (we started with 4)

L_3 has the largest itemset

C_4 is empty

Stop

Discovering Rules (from the full version of the paper)

Naïve algorithm:

- For every large itemset p
  - Find all non-empty subsets of p
  - For every subset q
    - Produce rule q → (p-q)
    - Accept if support(p) / support(q) >= minconf

Checking the subsets

- For efficiency, generate subsets using recursive DFS. If a subset q does not produce a rule, we do not need to check for subsets of q

Example

Given itemset : ABCD

If ABC → D does not have enough confidence

then AB → CD does not hold

Reason

For any subset q' of q:

Support(q') >= support(q)

$$
\begin{aligned}
& \text{confidence } (q' \rightarrow (p-q')) \\
= \; & \text{support}(p) / \text{support}(q') \\
\leq \; & \text{support}(p) / \text{support}(q) \\
= \; & \text{confidence } (q \rightarrow (p-q))
\end{aligned}
$$

Simple Algorithm

l = p
a = q

forall *large itemsets* l_k, $k \geq 2$ *do*

    genrules(l_k, l_k)

procedure genrules $(l_k$: *large k-itemset*, a_m: *large m-itemset*$)$

$A = \{(m-1)\text{-itemset } a_{m-1} \mid a_{m-1} \subset a_m\};$

forall $a_{m-1} \in A$ do begin

    $conf = support(l_k)/support(a_{m-1})$

    if $(conf \geq minconf)$ then begin

        output *the rule* $a_{m-1} \Rightarrow (l_k - a_{m-1})$;

        if $(m - 1 > 1)$ then

            call *genrules*(l_k, a_{m-1});

end

end

Check all the large itemsets

Check all the subsets

Check confidence of new rule

Output the rule

Continue the DFS over the subsets.

If there is no confidence the DFS branch cuts here

Faster Algorithm

- If (p-q) → q holds than all the rules
(p-q’) → q’ must hold
  - where q’ ⊆ q and is non-empty

Example:
If AB → CD holds,
then so do ABC → D and ABD → C

Idea
- Start with 1-item consequent and generate larger consequents
- If a consequent does not hold, do not look for bigger ones
- The candidate set will be a subset of the simple algorithm

Performance

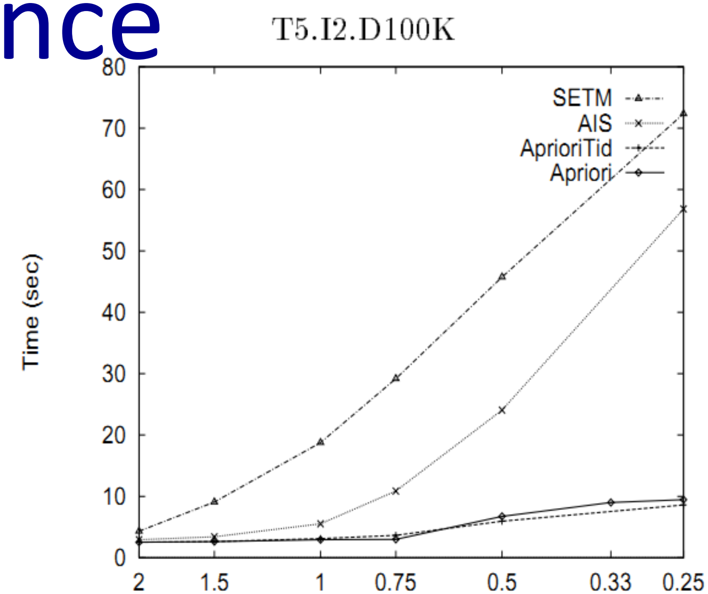
- Synthetic data modeling “real world”
  - People tend to buy things in sets
- Used the following parameters:

$\lvert D \rvert$	Number of transactions
$\lvert T \rvert$	Average size of the transactions
$\lvert I \rvert$	Average size of the maximal potentially large itemsets
$\lvert L \rvert$	Number of maximal potentially large itemsets
N	Number of items

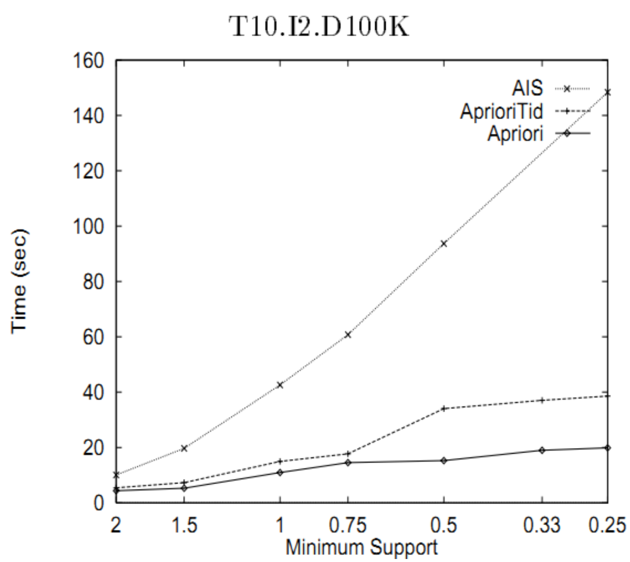
- The above are used in the names of the datasets: **T10I2D100K**
- Pick the size of the next transaction from a Poisson distribution with mean \|T\|
- Randomly pick determined large itemset and put in transaction, if too big overflow into next transaction

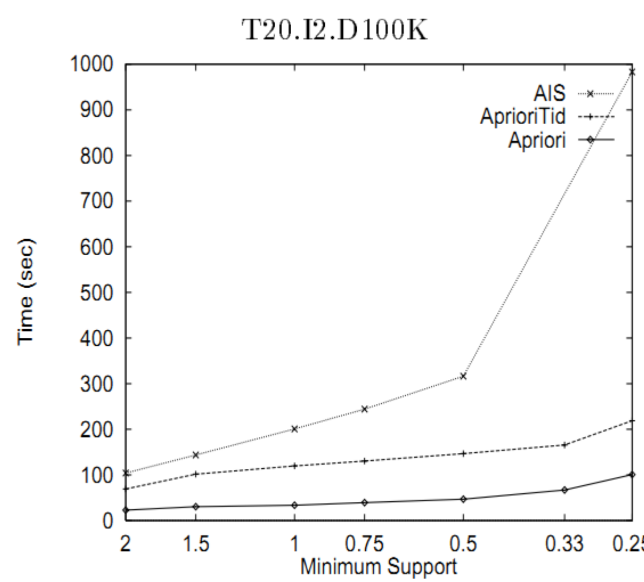
Performance

- Support decreases => time increases
- Apriori beats AIS and SETM
  - their candidate set is much larger
- AprioriTID is “almost” as good as Apriori, BUT Slower for larger problems



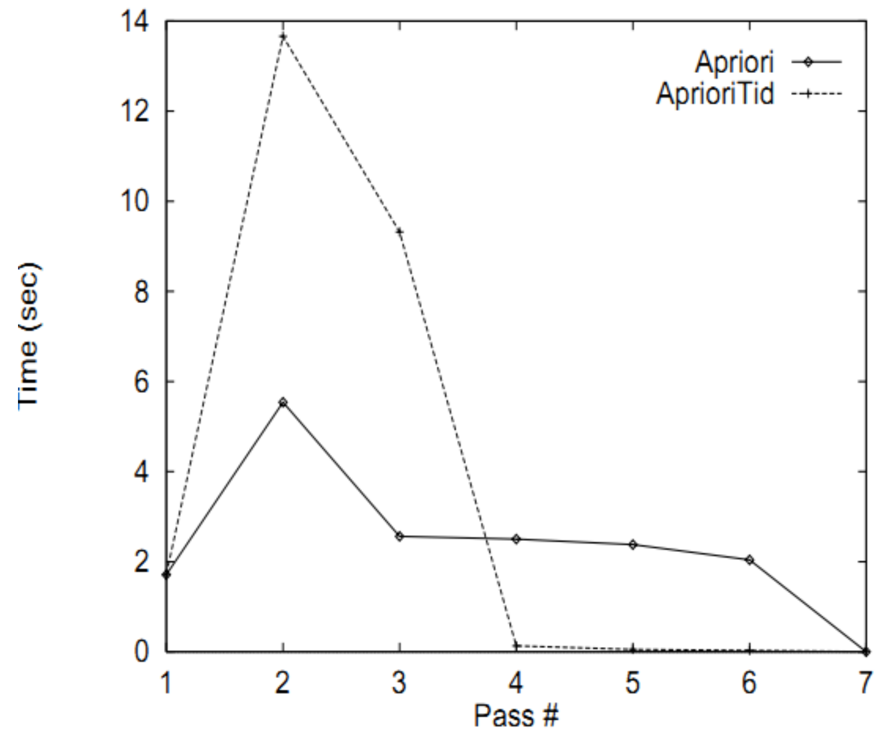
- C^*_k does not fit in memory and increases with #transactions





Performance

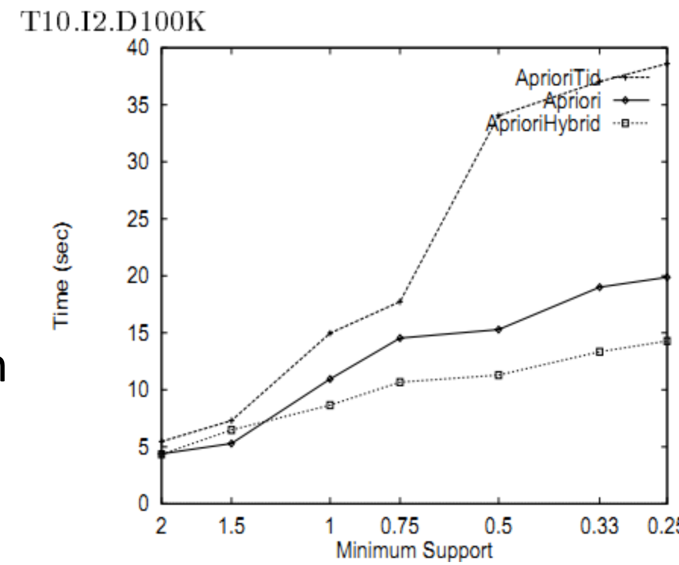
- AprioriTid is effective in later passes
  - Scans C^*_k instead of the original dataset
  - becomes small compared to original dataset

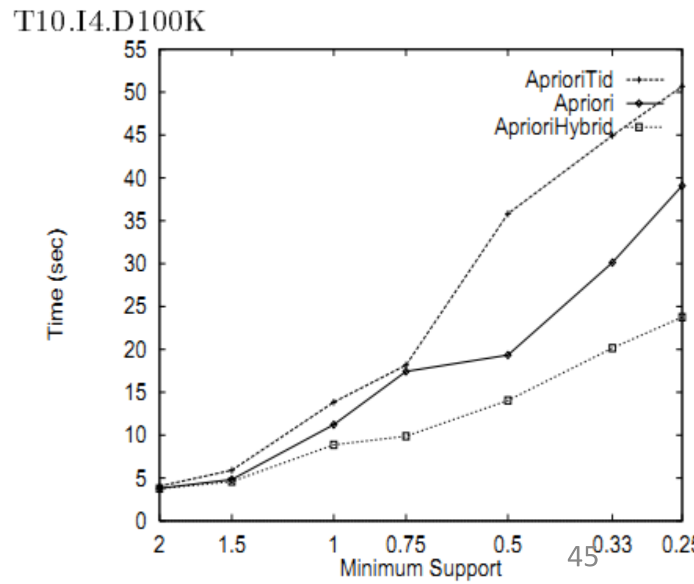


- When fits in memory, AprioriTid is faster than Apriori

AprioriHybrid

- Use Apriori in initial passes
- Switch to AprioriTid when it can fit in memory
  - estimate the size of C^*_k if it had been generated
  - $= \Sigma_{c \in Ck}$ support(c) + #transactions
  - if it fits in memort and fewer larger candidates in the current pass than previous pass, then switch
  - to avoid the case that C*k fits in the current pas but not in the next pass
- Switch happens at the end of the pass
  - Has some overhead to switch
- Relies on size drop
  - If switch happens late, will have slightly worse performance
- Still mostly better or as good as apriori





Summary

- Association rules are important
- This paper gives algorithms to find all association rules with required support and confidence
- Perform better than previous algorithms
- Scale well for large datasets