Decision Making for Robots and Autonomous Systems

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Utility Theory

Recall:
- We want to build rational agents
- Rational agents act to maximize performance

Performance is expressed using (equivalently):
- Object function
- Performance measure
- Performance metric
- Utility function

(presentation follows Ch. 3 in Kochenderfer)
Utility Theory

Most basic thing:

• Agent has to prefer some things over others.

Notion of preference over events:

• Prefer one event (or outcome) to another.
• Later: use these preferences to choose action.
Utility Theory

Some basic formal definitions of *preference*.

If we prefer B over A:

\[ A \prec B \]

If we have no preference between A and B:

\[ A \sim B \]

If we’re not sure if we prefer B over A or have no preference:

\[ A \preceq B \]
Utility theory

A lottery is defined by:

\[ \{(E_1, p_1); (E_2, p_2); \ldots (E_n; p_n)\} \]

... a set of events (over which we have a preference), each with an associated probability of occurrence.

Naturally:

\[ \sum_{i} p_i = 1 \]
Utility theory

Neumann-Morgenstern axioms - posited for *rational* prefs.

**Completeness:** only one of \( A \preceq B, B \preceq A, A \sim B \)

**Transitivity:** if \( A \preceq B \) and \( B \preceq C \) then \( A \preceq C \)

**Continuity:** if \( A \preceq B \preceq C \) then

\[
\exists p \quad \{(A, p), (C, (1 - p))\} \sim B
\]

**Independence:** if \( A \preceq B \)

\[
\forall C, p \quad \{(A, p), (C, (1 - p))\} \prec \{(B, p), (C, (1 - p))\}
\]
Utility theory

The von Neumann-Morgenstern axioms imply the existence of a real-valued utility measure, $U$.

There exists real-valued function $U$:

- $U : E \rightarrow \mathbb{R}$
- $U(A) < U(B)$ \iff $A \prec B$
- $U(A) = U(B)$ \iff $A \sim B$

$U$ is unique up to an affine transform:

$$U_2 = mU_1 + c$$

for positive $m$. 
Utility Theory

Now that we have a real-valued $U$ …

$$U\left(\{(E_1, p_1); \ldots (E_n, p_n)\}\right) = \sum_i p_i U(E_i)$$

… expected utility of a lottery. This assigns an explicit utility value to an uncertain event.
The Maximum Expected Utility Principle

Every action results in a lottery.

The agent should select the outcome that leads to the lottery with maximum expected utility.

Let’s say each action $a$ leads to lottery outcome $o$ with probability $P(o | a)$. The agent should select:

$$a^* = \max_a \sum_o P(o|a)U(o)$$

Variations of this are the whole core of rationality.
MEUP

What assumptions does this make?
Humans are Irrational

Would you prefer:

• A: 100% change of losing 75 lives.
• B: 80% chance of losing 100 lives.

Most prefer B over A: \( U(\text{lose 75}) < 0.8U(\text{lose 100}) \)

Now:

• C: 10% chance of losing 75 lives.
• D: 8% chance of losing 100 lives.

Most prefer C over D: \( 0.1U(\text{lose 75}) > 0.08U(\text{lose 100}) \)

[Tversky and Kahneman]
Humans are Irrational

Certainty effect:
- Humans exaggerate certain losses vs. probable losses.
- Exaggerate certain gains vs. probable gains.
Framing Effect

Two ways of phrasing the question:

Epidemic in town of 600 people.

E: 200 people will be saved.
F: 1/3 chance that 600 will be saved, 2/3 chance nobody saved.

People mostly choose E over F.

G: 400 people will die.
H: 1/3 chance that nobody dies, 2/3 chance that 600 die.

Majority if students choose H over G.
Prospect Theory

Leads to *prospect theory* for human decision-making:
- Set frame point.
- Utilities re-arranged around losses vs. gains.
The Value of Money

Risk neutral vs. averse vs. seeking
Assumptions

Thoughts on the assumptions here:

• The agent \textbf{knows} its utility function.
• The agent \textbf{knows} \(U(E)\), for all events \(E\).
• The agent has \textbf{just one} utility function.
• That utility function is \textbf{stationary}.

Thursday we look at the case where the agent does not know \(U(E)\) - and must learn it.