Relational Database Design Theory

Introduction to Databases

CompSci 316 Fall 2019



Announcements (Mon. Sep. 9)

- Gradiance ER due this Wednesday
- Gradiance FD and MVD assigned
- Homework 1 due next Monday (11:59pm)
 - RA debugger for Problem 1 available at https://ratest.cs.duke.edu/
- Course project description posted
 - Read it!
 - "Mixer" in a week and a half
 - Milestone 1 right after fall break
 - Teamwork required: 5 people per team on average

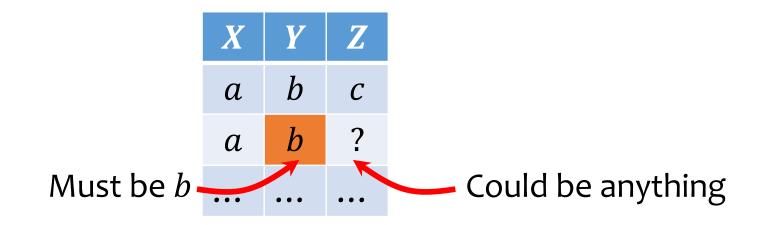
Motivation

uid	uname	gid
142	Bart	dps
123	Milhouse	gov
857	Lisa	abc
857	Lisa	gov
456	Ralph	abc
456	Ralph	gov
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- Why is UserGroup (<u>uid</u>, uname, <u>gid</u>) a bad design?
 - Leads to _____ anomalies
- Wouldn't it be nice to have a systematic approach to detecting and removing redundancy in designs?
 - Dependencies, decompositions, and normal forms

Functional dependencies

- A functional dependency (FD) has the form $X \rightarrow Y$, where X and Y are sets of attributes in a relation R
- $X \rightarrow Y$ means that whenever two tuples in R agree on all the attributes in X, they must also agree on all attributes in Y



FD examples

Address (street_address, city, state, zip)

- street_address, city, state \rightarrow zip
- $zip \rightarrow city$, state
- zip, state \rightarrow zip?
 - This is a trivial FD
 - Trivial FD: LHS \supseteq RHS
- $zip \rightarrow state, zip$?
 - This is non-trivial, but not completely non-trivial
 - Completely non-trivial FD: LHS \cap RHS = Ø

Redefining "keys" using FD's

A set of attributes *K* is a key for a relation *R* if

- $K \rightarrow \text{all (other)}$ attributes of R
 - That is, *K* is a "super key"
- No proper subset of K satisfies the above condition
 - That is, *K* is minimal

Reasoning with FD's

Given a relation R and a set of FD's \mathcal{F}

- Does another FD follow from \mathcal{F} ?
 - Are some of the FD's in \mathcal{F} redundant (i.e., they follow from the others)?
- Is *K* a key of *R*?
 - What are all the keys of *R*?

Attribute closure

- Given R, a set of FD's \mathcal{F} that hold in R, and a set of attributes Z in R: The closure of Z (denoted Z^+) with respect to \mathcal{F} is the set of all attributes $\{A_1, A_2, ...\}$ functionally determined by Z (that is, $Z \rightarrow A_1A_2$...)
- Algorithm for computing the closure
 - Start with closure = Z
 - If $X \rightarrow Y$ is in \mathcal{F} and X is already in the closure, then also add Y to the closure
 - Repeat until no new attributes can be added

A more complex example

UserJoinsGroup (uid, uname, twitterid, gid, fromDate) Assume that there is a 1-1 correspondence between our users and Twitter accounts

- uid \rightarrow uname, twitterid
- twitterid \rightarrow uid
- uid, gid \rightarrow fromDate

Not a good design, and we will see why shortly

Example of computing closure

- {gid, twitterid}⁺ = ?
- twitterid \rightarrow uid
 - Add uid
 - Closure grows to { gid, twitterid, uid }
- uid \rightarrow uname, twitterid
 - Add uname, twitterid
 - Closure grows to { gid, twitterid, uid, uname }
- uid, gid \rightarrow fromDate
 - Add fromDate
 - Closure is now all attributes in UserJoinsGroup

 \mathcal{F} includes: uid \rightarrow uname, twitterid twitterid \rightarrow uid uid, gid \rightarrow fromDate

Using attribute closure

Given a relation R and set of FD's \mathcal{F}

- Does another $FD X \rightarrow Y$ follow from \mathcal{F} ?
 - Compute X^+ with respect to \mathcal{F}
 - If $Y \subseteq X^+$, then $X \to Y$ follows from \mathcal{F}
- Is K a key of R?
 - Compute K^+ with respect to \mathcal{F}
 - If K^+ contains all the attributes of R, K is a super key
 - Still need to verify that *K* is minimal (how?)

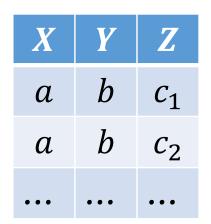
Rules of FD's

- Armstrong's axioms
 - Reflexivity: If $Y \subseteq X$, then $X \to Y$
 - Augmentation: If $X \rightarrow Y$, then $XZ \rightarrow YZ$ for any Z
 - Transitivity: If $X \to Y$ and $Y \to Z$, then $X \to Z$
- Rules derived from axioms
 - Splitting: If $X \to YZ$, then $X \to Y$ and $X \to Z$
 - Combining: If $X \to Y$ and $X \to Z$, then $X \to YZ$

Using these rules, you can prove or disprove an FD given a set of FDs

Non-key FD's

- Consider a non-trivial FD $X \rightarrow Y$ where X is not a super key
 - Since X is not a super key, there are some attributes (say Z) that are not functionally determined by X



That *a* should be mapped to *b* is recorded multiple times: redundancy, update/insertion/deletion anomaly

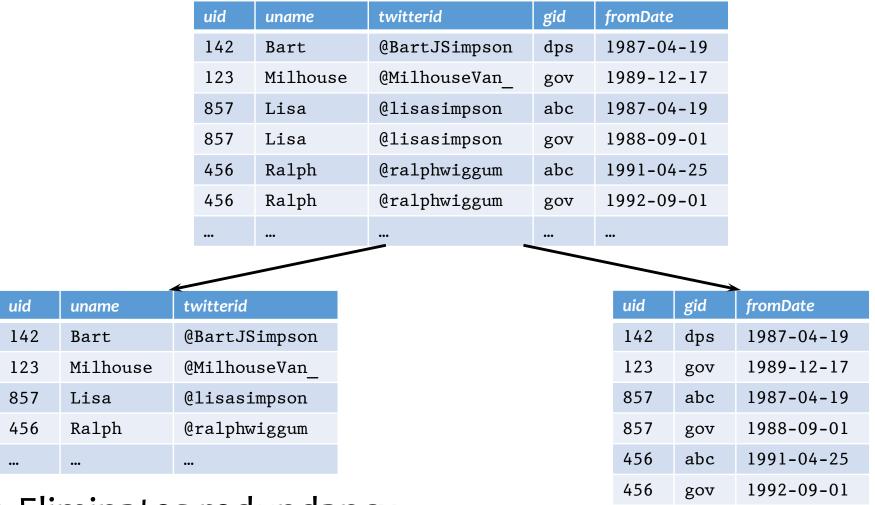
Example of redundancy

UserJoinsGroup (uid, uname, twitterid, gid, fromDate)

- uid \rightarrow uname, twitterid
- (... plus other FD's)

uid	uname	twitterid	gid	fromDate
142	Bart	@BartJSimpson	dps	1987-04-19
123	Milhouse	@MilhouseVan_	gov	1989-12-17
857	Lisa	@lisasimpson	abc	1987-04-19
857	Lisa	@lisasimpson	gov	1988-09-01
456	Ralph	@ralphwiggum	abc	1991-04-25
456	Ralph	@ralphwiggum	gov	1992-09-01

Decomposition

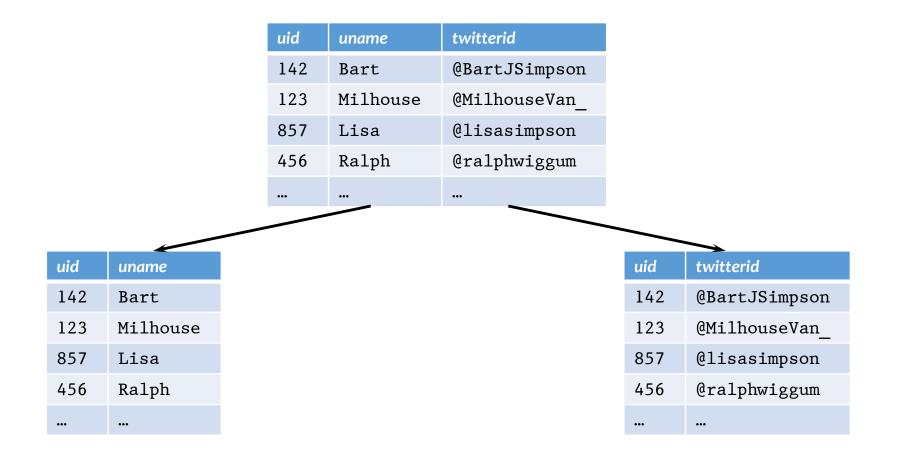


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- Eliminates redundancy
- To get back to the original relation:



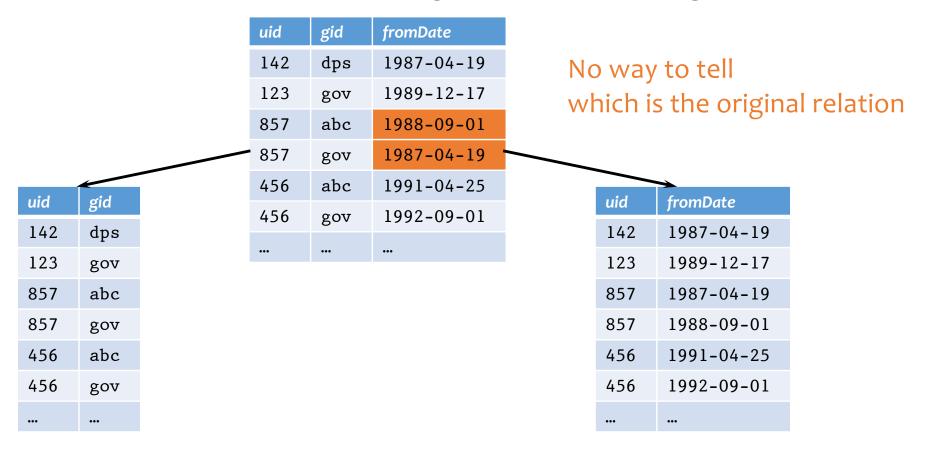
		uid	gid	fromDate		
		142	dps	1987-04-19		
		123	gov	1989-12-17		
		857	abc	1987-04-19		
		857	gov	1988-09-01		
		456	abc	1991-04-25		
		456	gov	1992-09-01	-	
uid	gid					uid
42	dps					142
23	gov					123
7	abc					857
357	gov					857
56	abc					456
56	gov					456
						•••

Lossless join decomposition

- Decompose relation R into relations S and T
 - $attrs(R) = attrs(S) \cup attrs(T)$
 - $S = \pi_{attrs(S)}(R)$
 - $T = \pi_{attrs(T)}(R)$
- The decomposition is a lossless join decomposition if, given known constraints such as FD's, we can guarantee that $R = S \bowtie T$
- Any decomposition gives $R \subseteq S \bowtie T$ (why?)
 - A lossy decomposition is one with $R \subset S \bowtie T$

Loss? But I got more rows!

- "Loss" refers not to the loss of tuples, but to the loss of information
 - Or, the ability to distinguish different original relations



Questions about decomposition

- When to decompose
- How to come up with a correct decomposition (i.e., lossless join decomposition)

An answer: BCNF

- A relation *R* is in Boyce-Codd Normal Form if
 - For every non-trivial FD $X \rightarrow Y$ in R, X is a super key
 - That is, all FDs follow from "key → other attributes"
- When to decompose
 - As long as some relation is not in BCNF
- How to come up with a correct decomposition
 - Always decompose on a BCNF violation (details next)
 Then it is guaranteed to be a lossless join decomposition!

BCNF decomposition algorithm

- Find a BCNF violation
 - That is, a non-trivial FD $X \rightarrow Y$ in R where X is not a super key of R
- Decompose *R* into *R*₁ and *R*₂, where
 - R_1 has attributes $X \cup Y$
 - R_2 has attributes $X \cup Z$, where Z contains all attributes of R that are in neither X nor Y
- Repeat until all relations are in BCNF

BCNF decomposition example

uid \rightarrow uname, twitterid twitterid \rightarrow uid uid, gid \rightarrow fromDate

UserJoinsGroup (uid, uname, twitterid, gid, fromDate)

BCNF violation: uid \rightarrow uname, twitterid

User (uid, uname, twitterid)

uid \rightarrow uname, twitterid twitterid \rightarrow uid

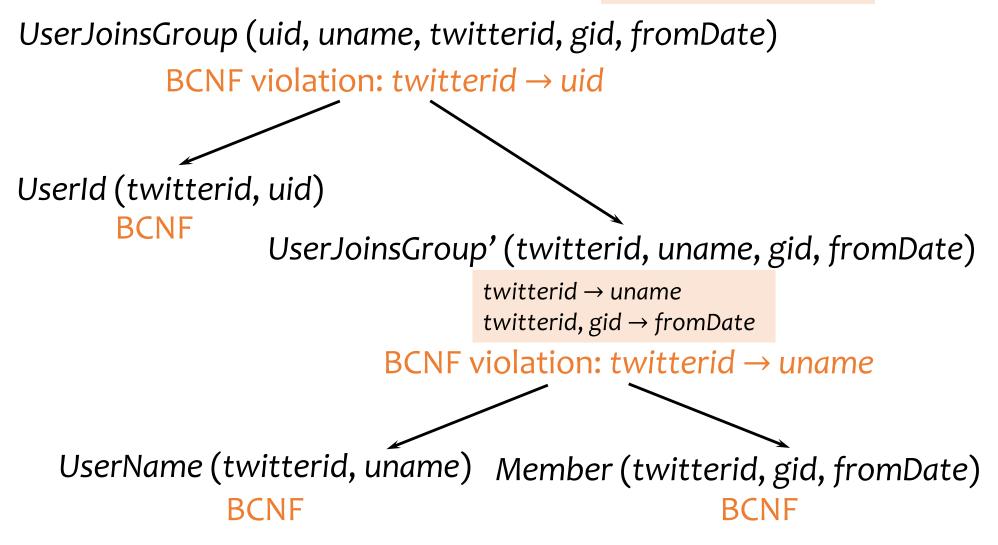
BCNF

Member (uid, gid, fromDate) uid, gid \rightarrow fromDate

BCNF

Another example

uid \rightarrow uname, twitterid twitterid \rightarrow uid uid, gid \rightarrow fromDate



Why is BCNF decomposition lossless

Given non-trivial $X \rightarrow Y$ in R where X is not a super key of R, need to prove:

- Anything we project always comes back in the join: $R \subseteq \pi_{XY}(R) \bowtie \pi_{XZ}(R)$
 - Sure; and it doesn't depend on the FD
- Anything that comes back in the join must be in the original relation:

 $R \supseteq \pi_{XY}(R) \bowtie \pi_{XZ}(R)$

• Proof will make use of the fact that $X \rightarrow Y$

Recap

- Functional dependencies: a generalization of the key concept
- Non-key functional dependencies: a source of redundancy
- BCNF decomposition: a method for removing redundancies
 - BNCF decomposition is a lossless join decomposition
- BCNF: schema in this normal form has no redundancy due to FD's

BCNF = no redundancy?

- User (uid, gid, place)
 - A user can belong to multiple groups
 - A user can register places she's visited
 - Groups and places have nothing to do with other
 - FD's?
 - BCNF?
 - Redundancies?

uid	gid	place
142	dps	Springfield
142	dps	Australia
456	abc	Springfield
456	abc	Morocco
456	gov	Springfield
456	gov	Morocco

Multivalued dependencies

- X → Y means that whenever two rows in R agree on all the attributes of X, then we can swap their Y components and get two rows that are also in R

X	Y	Z
а	b_1	<i>C</i> ₁
а	<i>b</i> ₂	<i>C</i> ₂
а	<i>b</i> ₂	<i>c</i> ₁
а	<i>b</i> ₁	<i>C</i> ₂
•••	•••	•••

MVD examples

User (uid, gid, place)

- uid \rightarrow gid
- uid \rightarrow place
 - Intuition: given uid, gid and place are "independent"
- uid, gid \rightarrow place
 - Trivial: LHS U RHS = all attributes of *R*
- uid, gid → uid
 - Trivial: LHS \supseteq RHS

Complete MVD + FD rules

- FD reflexivity, augmentation, and transitivity
- MVD complementation: If $X \rightarrow Y$, then $X \rightarrow attrs(R) - X - Y$
- MVD augmentation: If $X \rightarrow Y$ and $V \subseteq W$, then $XW \rightarrow YV$
- MVD transitivity: If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z - Y$
- Replication (FD is MVD): If $X \rightarrow Y$, then $X \rightarrow Y$

Try proving things using these!?

• Coalescence: If $X \rightarrow Y$ and $Z \subseteq Y$ and there is some W disjoint from Y such that $W \rightarrow Z$, then $X \rightarrow Z$

An elegant solution: chase

- Given a set of FD's and MVD's \mathcal{D} , does another dependency d (FD or MVD) follow from \mathcal{D} ?
- Procedure
 - Start with the premise of *d*, and treat them as "seed" tuples in a relation
 - Apply the given dependencies in ${\mathcal D}$ repeatedly
 - If we apply an FD, we infer equality of two symbols
 - If we apply an MVD, we infer more tuples
 - If we infer the conclusion of *d*, we have a proof
 - Otherwise, if nothing more can be inferred, we have a counterexample

Proof by chase

• In R(A, B, C, D), does $A \rightarrow B$ and $B \rightarrow C$ imply that $A \rightarrow C?$

•	Have:	A	B	С	D	Need:	A	B	С	D	
		а	b_1	<i>c</i> ₁	d_1		а	b_1	<i>C</i> ₂	d_1	er f
		а	b_2	<i>C</i> ₂	d_2		а	b_2	<i>C</i> ₁	d_2	er s
	$A \twoheadrightarrow B$	а	<i>b</i> ₂	<i>c</i> ₁	d_1						
	A → D	а	b_1	<i>C</i> ₂	d_2						
	$B \twoheadrightarrow C$	а	<i>b</i> ₂	<i>c</i> ₁	d_2						
	D C	а	<i>b</i> ₂	<i>C</i> ₂	d_1						
	$B \twoheadrightarrow C$	а	b_1	<i>C</i> ₂	d_1						
	D C	а	b_1	<i>c</i> ₁	d_2						

Another proof by chase

• In R(A, B, C, D), does $A \rightarrow B$ and $B \rightarrow C$ imply that $A \rightarrow C$?

Have: $A \ B \ C \ D$ $a \ b_1 \ c_1 \ d_1$ $a \ b_2 \ c_2 \ d_2$ $A \rightarrow B$ $B \rightarrow C$ $b_1 = b_2$ $b_1 = c_2$ Need: $c_1 = c_2$

In general, with both MVD's and FD's, chase can generate both new tuples and new equalities

Counterexample by chase

• In R(A, B, C, D), does $A \rightarrow BC$ and $CD \rightarrow B$ imply that $A \rightarrow B$?

Have:	A	B	<i>C</i>	D
	а	b_1	<i>c</i> ₁	d_1
	а	b_2	<i>C</i> ₂	d_2
$A \twoheadrightarrow BC$	а	b_2 b_1	<i>C</i> ₂	d_1
A → DL	а	b_1	C_1	d_2

Counterexample!

Need:

$$b_1 = b_2$$

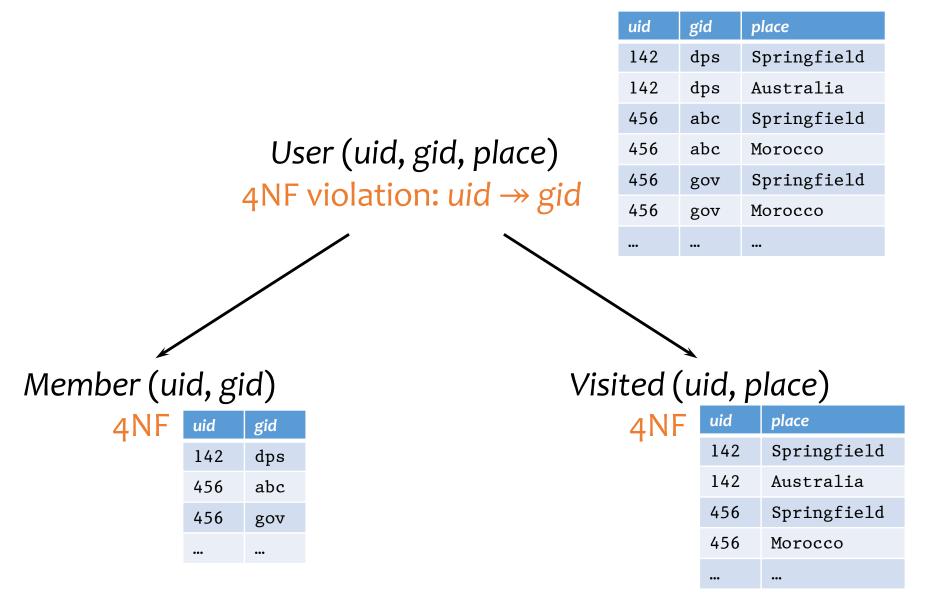
4NF

- A relation *R* is in Fourth Normal Form (4NF) if
 - For every non-trivial MVD $X \rightarrow Y$ in R, X is a superkey
 - That is, all FD's and MVD's follow from "key → other attributes" (i.e., no MVD's and no FD's besides key functional dependencies)
- 4NF is stronger than BCNF
 - Because every FD is also a MVD

4NF decomposition algorithm

- Find a 4NF violation
 - A non-trivial MVD $X \rightarrow Y$ in R where X is not a superkey
- Decompose *R* into *R*₁ and *R*₂, where
 - R_1 has attributes $X \cup Y$
 - R_2 has attributes $X \cup Z$ (where Z contains R attributes not in X or Y)
- Repeat until all relations are in 4NF
- Almost identical to BCNF decomposition algorithm
- Any decomposition on a 4NF violation is lossless

4NF decomposition example



Summary

- Philosophy behind BCNF, 4NF: Data should depend on the key, the whole key, and nothing but the key!
 - You could have multiple keys though
- Other normal forms
 - 3NF: More relaxed than BCNF; will not remove redundancy if doing so makes FDs harder to enforce
 - 2NF: Slightly more relaxed than 3NF
 - 1NF: All column values must be atomic

