Problem 1 (20 points)

Represent the above directed graph
(a) by an adjacency matrix.
(b) by a linked adjacency list.

Problem 2 (25 points)

Given a bipartite graph $G$ and a matching $M$, the following two statements are equivalent:
(1) The current matching is maximum.
(2) There are no augmenting paths.
Consider the bipartite graph above. The thick edges indicate a maximum matching. Show that there are no augmenting paths.
Problem 3 (30 points)

(a) Draw the depth-first search forest. (Assume we start from node a, and explore edges from left to right.)

(b) Find the strongly connected components.

(c) Suppose the above graph is undirected, draw the breadth-first search tree. (Assume we start from node a, and explore edges from left to right.)

Problem 4 (25 points)

List the order of nodes in a Topological Sort of the graph.

Extra Credit (20 points)

Let $G = (V, E)$ be a complete graph, that is, a graph in which there is an edge between every pair of distinct vertices. Let $G' = (V, E')$ be a directed graph in which $E'$ is $E$ with each edge given an arbitrary orientation. Show that $G'$ has a directed path that includes every vertex exactly once.