CPS 130 Homework Solutions

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HOMEWORK 1

1.1: Following the steps introduced in lecture 1 for Russian Peasant’s Algorithm.

1.2: (a)

int Exp(int a, int b)
{
    int c;

    if (b == 0)
        return 1;
    if (b % 2 == 1) {
        c = Exp(a, (b-1) >> 1);
        return c * c * a;
    }
    if (b % 2 == 0) {
        c = Exp(a, b >> 1);
        return c * c;
    }
}

(b)
1. When b is 0, Exp(a, b) = 1 = a^0.
2. Assume for any nonegative integer b < b, Exp(a, b) = a^b holds. If b is odd, c = Exp(a, (b - 1) >> 1) = a^{b-1}\text{>>}1 because of assumption, Exp(a, b) = c * c * a = a^{b-1}\text{>>}1 * a = a^b; If b is even, c = Exp(a, b >> 1) = a^{b\text{>>}1} because of assumption, Exp(a, b) = c * c = a^{b\text{>>}1} * a^{b\text{>>}1} = a^b.
3. b decreases on each recursion down to zero. The algorithm will terminate.

(c) b decreases by bit shifting by 1 on each recursion, so the number of recursive calls is Θ(lg b).
1.4:
1. When $n$ is 1, $T(n) = 2^{1+|\log(n)|} - 1 = 1$.
2. Assume for any nonnegative integer $n' < n$, $T(n') = 2^{1+|\log(n')|} - 1$ holds.
3. For $n$,

$$T(n) = 1 + 2T\left(\left\lfloor n/2 \right\rfloor\right)$$
$$= 1 + 2\left(2^{1+|\log(\left\lfloor n/2 \right\rfloor)|} - 1\right), \text{ by assumption}$$
$$= 1 + 2\left(2^{1+|\log(n/2)|} - 1\right)$$
$$= 1 + 2^{2+|\log(n/2)|} - 2$$
$$= 2^{2+|\log(n)|-1} - 1$$
$$= 2^{2+|\log(n)|-1} - 1$$
$$= 2^{1+|\log(n)|} - 1$$

1.5: The first statement is true, because $2^{n+1} = 2 \cdot 2^n$; The second one is not true, because $2^{2n} = (2^n)^2 = \omega(2^n)$.

1.6: $\log^* n, 2^{\log_2 n}, n \log n, n^3, (\log n)!, n^{\log\log n}, 2^n, e^n, n!$