Solving Problems Recursively

- **Recursion is an indispensable tool in a programmer’s toolkit**
  - Allows many complex problems to be solved simply
  - Elegance and understanding in code often leads to better programs: easier to modify, extend, verify (and sometimes more efficient!! See expotest.cpp)
  - Sometimes recursion isn’t appropriate, when it’s bad it can be very bad---every tool requires knowledge and experience in how to use it

- **The basic idea is to get help solving a problem from coworkers (clones) who work and act like you do**
  - Ask clone to solve a simpler but similar problem
  - Use clone’s result to put together your answer

- **Need both concepts: call on the clone and use the result**
Fundamentals of Recursion

- **Base case (aka exit case)**
  - Simple case that can be solved with no further computation
  - Does not make a recursive call

- **Reduction step (aka Inductive hypothesis)**
  - Reduce the problem to another *smaller* one of the same structure
  - Make a recursive call, with some parameter or other measure that decreases or moves towards the base case
    - Ensure that sequence of calls eventually reaches the base case
    - “Measure” can be tricky, but usually it’s straightforward

- **The Leap of Faith!**
  - If it works for the reduction step is correct and there is proper handling of the base case, the recursion is correct.

- **What row are you in?**
Recursive example

```cpp
void WriteBinary(int n) {
    // Writes the binary representation of n to cout.
    if (n < 0) {
        cout << "-";
        WriteBinary(-n);
    } else if (n < 2) {
        cout << n;
    } else {
        WriteBinary(n/2);
        cout << n % 2;
    }
}
```
Recursive example

```c++
void mystery(int n)
{
    if (n <= 1)
        cout << n;
    else {
        mystery(n/2);
        cout << "," << n;
    }
}
```
Classic examples of recursion

• For some reason, computer science uses these examples:
  - Factorial: we can use a loop or recursion (see `facttest.cpp`), is this an issue?
  - Fibonacci numbers: 1, 1, 2, 3, 5, 8, 13, 21, ...
    • $F(n) = F(n-1) + F(n-2)$, why isn’t this enough? What’s needed?
    • Classic example of bad recursion, to compute $F(6)$, the sixth Fibonacci number, we must compute $F(5)$ and $F(4)$. What do we do to compute $F(5)$? Why is this a problem?
  - Towers of Hanoi
    • N disks on one of three pegs, transfer all disks to another peg, never put a disk on a smaller one, only on larger
    • Every solution takes “forever” when N, number of disks, is large
Towers of Hanoi

- The origins of the problem/puzzle may be in the far east
  - Move n disks from one peg to another in a set of three

```cpp
void Move(int from, int to, int aux, int numDisks)
// pre: numDisks on peg # from,
// move to peg # to
// post: disks moved from peg 'from'
//       to peg 'to' via 'aux'
{
    if (numDisks == 1)
    { cout << "move " << from << " to " << to << endl;
    }
    else
    { Move (from,aux,to, numDisks - 1);
      Move (from,to,aux, 1);
      Move (aux,to,from, numDisks - 1);
    }
}
```
Fibonacci: Don’t do this recursively

```c
long RecFib(int n)
    // precondition: 0 <= n
    // postcondition: returns the n-th Fibonacci number
{
    if (0 == n || 1 == n)
    { return 1; }
    else
    { return RecFib(n-1) + RecFib(n-2); }
}
```

• How many clones/calls to compute F(5)?

```
// How many calls of F(1)?
// How many total calls?
See recfib2.cpp for caching code
```

A Computer Science Tapestry
Print words entered, but backwards

- Can use a vector, store all the words and print in reverse order
  - The vector is probably the best approach, but recursion works too

```cpp
void PrintReversed()
{
    string word;
    if (cin >> word)  // reading succeeded?
    {
        PrintReversed();  // print the rest reversed
        cout << word << endl;  // then print the word
    }
}

int main()
{
    PrintReversed();
}
```

- The function `PrintReversed` reads a word, prints the word only after the clones finish printing in reverse order
  - Each clone has its own version of the code, its own `word` variable
Exponentiation

- Computing $x^n$ means multiplying $n$ numbers (or does it?)
  - What’s the easiest value of $n$ to compute $x^n$?
  - If you want to multiply only once, what can you ask a clone?

```cpp
double Power(double x, int n)
// post: returns $x^n$
{
    if (n == 0)
    {
        return 1.0;
    }
    return x * Power(x, n-1);
}
```

- What about an iterative version?
Faster exponentiation

- **How many recursive calls are made to compute** $2^{1024}$?
  - How many multiplies on each call? Is this better?

```c
double Power(double x, int n)
// post: returns x^n
{
    if (n == 0)
    {
        return 1.0;
    }
    double semi = Power(x, n/2);
    if (n % 2 == 0)
    {
        return semi*semi;
    }
    return x * semi * semi;
}
```

- **What about an iterative version of this function?**
Recursive and Iterative log powers

- In the program expotest.cpp we calculate $x^n$ using $\log(n)$ multiplications (basically). We do this both iteratively and recursively using BigInt variables
  - We saw the iterative code in Chapter 5 with doubles
  - BigInt has overloaded operators so these values work like ints and doubles
  - We use the CTimer class to time the difference in using these functions (ctimer.h)

- The recursive version is faster, sometimes much faster
  - Using doubles we wouldn’t notice a difference
  - Artifact of algorithm? Can we “fix the problem”?
  - Natural version of both in programs, optimizing tough.
What’s better: recursion/iteration?

- There’s no single answer, many factors contribute
  - Ease of developing code assuming other factors ok
  - Efficiency (runtime or space) can matter, but don’t worry about efficiency unless you know you have to
- In some examples, like Fibonacci numbers, recursive solution does extra work, we’d like to avoid the extra work
  - Iterative solution is efficient
  - The recursive inefficiency of “extra work” can be fixed if we remember intermediate solutions: static variables

- Static function variable: maintain value over all function calls
  - Local variables constructed each time function called
Fixing recursive Fibonacci: *recfib2.cpp*

```cpp
long RecFib(int n)
// precondition: 0 <= n <= 30
// postcondition: returns the n-th Fibonacci number
{
    static tvector<int> storage(31,0);

    if (0 == n || 1 == n)    return 1;
    else if (storage[n] != 0) return storage[n];
    else {
        storage[n] = RecFib(n-1) + RecFib(n-2);
        return storage[n];
    }
}
```

- **What does storage do? Why initialize to all zeros?**
  - Static variables initialized first time function called
  - Maintain values over calls, not reset or re-initialized
Thinking recursively

- **Problem:** find the largest element in a vector
  - **Iteratively:** loop, remember largest seen so far
  - **Recursive:** find largest in \([1..n)\), then compare to 0th element

```cpp
double Max(const tvector<double>& a)
// pre: a contains a.size() elements, 0 < a.size()
// post: return maximal element of a
{
    int k;
    double max = a[0];
    for (k = 0; k < a.size(); k++)
    {
        if (max < a[k]) max = a[k];
    }
    return max;
}
```

- In a recursive version what is base case, what is measure of problem size that decreases (towards base case)?
Recursive Max

double RecMax(const tvector<double>& a, int first)
// pre: a contains a.size() elements, 0 < a.size()
//      first < a.size()
// post: return maximal element a[first..size()-1]
{
    if (first == a.size()-1) // last element, done
    {
        return a[first];
    }
    double maxAfter = RecMax(a,first+1);
    if (maxAfter < a[first]) return a[first];
    else return maxAfter;
}

● What is base case (conceptually)?
● We can use RecMax to implement Max as follows
    return RecMax(a,0);
Recognizing recursion:

```c
void Change(tvector<int>& a, int first, int last)
// post: a is changed
{
    if (first < last)
    {
        int temp = a[first];  // swap a[first], a[last]
        a[first] = a[last];
        a[last] = temp;
        Change(a, first+1, last-1);
    }
}
// original call (why?): Change(a, 0, a.size()-1);
```

● What is base case? (no recursive calls)
● What happens before recursive call made?
● How is recursive call closer to the base case?
More recursion recognition

```cpp
int Value(const tvector<int>& a, int index)
   // pre: ??
   // post: a value is returned
{
   if (index < a.size())
   {
      return a[index] + Value(a,index+1);
   }
   return 0;
}
// original call:  cout << Value(a,0) << endl;
```

- What is base case, what value is returned?
- How is progress towards base case realized?
- How is recursive value used to return a value?
- What if a is vector of doubles, does anything change?
One more recognition

```c
void Something(int n, int& rev)
// post: rev has some value based on n
{
    if (n != 0)
    {
        rev = (rev*10) + (n % 10);
        Something(n/10, rev);
    }
}

int Number(int n)
{
    int value = 0;
    Something(n,value);
    return value;
}
```

- **What is returned by** `Number(13)`? `Number(1234)`?
  - This is a tail recursive function, last statement recursive
  - Can turn tail recursive functions into loops very easily
Non-recursive version

```c
int Number(int n)
   // post: return reverse of n, e.g., 4231 for n==1234
{
    int rev = 0;    // rev is reverse of n's digits so far
    while (n != 0)
    {
        rev = (rev * 10) + n % 10;
        n /= 10;
    }
}
```

- Why did recursive version need the helper function?
  - Where does initialization happen in recursion?
  - How is helper function related to idea above?
- Is one of these easier to understand?
- What about developing rather than recognizing?
Blob Counting: Recursion at Work

- Blob counting is similar to what’s called Flood Fill, the method used to fill in an outline with a color (use the paint-can in many drawing programs to fill in)
  - Possible to do this iteratively, but hard to get right
  - Simple recursive solution

- Suppose a slide is viewed under a microscope
  - Count images on the slide, or blobs in a gel, or ...
  - Erase noise and make the blobs more visible

- To write the program we’ll use a class CharBitMap which represents images using characters
  - The program blobs.cpp and class Blobs are essential too
Counting blobs, the first slide

prompt> blobs
enter row col size 10 50
# pixels on: between 1 and 500: 200

| * * * * * *** * **** * * |
| * * *** ** ** * * * * * * |
| * *** * *** * * * * * * * * |
| * ** * * *** * * * * * * * * |
| * * ***** *** * * ** *** * |
| * * * * * * * * * * * * * * |
| * * *** * * * * * * * * * |
| * * ** * * * *** * * * * * |
| **** * * ** **** * *** * * * |
| ** * * * * * *** * * * * * * |

● How many blobs are there? Blobs are connected horizontally and vertically, suppose a minimum of 10 cells in a blob

➢ What if blob size changes?
Identifying Larger Blobs

blob size (0 to exit) between 0 and 50: 10

# blobs = 3

- The class Blob makes a copy of the CharBitMap and then counts blobs in the copy, by erasing noisy data (essentially)
  - In identifying blobs, too-small blobs are counted, then uncounted by erasing them
Identifying smaller blobs

blob size (0 to exit) between 0 and 50: 5

......1...............2................................
......1.1...........222................................
......111............333.2..............................4.

..............33.22......................................444......5.
..............33333.222.................................6.444......55.
..............2.........................6........444......555..

..............222.....77.................................6.444......555..

...8.............2...7..........................666.66........
..8888...........22..7777.........................666........
88...8...............77.7777............................

# blobs = 8

- What might be a problem if there are more than nine blobs?
  - Issues in looking at code: how do language features get in the way of understanding the code?
  - How can we track blobs, e.g., find the largest blob?
Issues that arise in studying code

- **What does static mean, values defined in Blobs?**
  - Class-wide values rather than stored once per object
  - All Blob variables would share `PIXEL_OFF`, unlike `myBlobCount` which is different in every object
  - When is static useful?

- **What is the class tmatrix?**
  - Two-dimensional vector, use `a[0][1]` instead of `a[0]`
  - First index is the row, second index is the column

- **We’ll study these concepts in more depth, a minimal understanding is needed to work on blobs.cpp**
Recursive helper functions

- Client programs use `Blobs::FindBlobs` to find blobs of a given size in a `CharBitMap` object
  - This is a recursive function, private data is often needed/used in recursive member function parameters
  - Use a helper function, not accessible to client code, use recursion to implement member function

- To find a blob, look at every pixel, if a pixel is part of a blob, identify the entire blob by sending out recursive clones/scouts
  - Each clone reports back the number of pixels it counts
  - Each clone “colors” the blob with an identifying mark
  - The mark is used to avoid duplicate (unending) work
Conceptual Details of BlobFill

- Once a blob pixel is found, four recursive clones are “sent out” looking horizontally and vertically, reporting pixel count
  - How are pixel counts processed by clone-sender?
  - What if all the clones ultimately report a blob that’s small?

- In checking horizontal/vertical neighbors what happens if there aren’t four neighbors? Is this a potential problem?
  - Who checks for valid pixel coordinates, or pixel color?
  - Two options: don’t make the call, don’t process the call

- Non-recursive member function takes care of looking for blobsign, then filling/counting/unfilling blobs
  - How is unfill/uncount managed?