Many problems require an approach similar to solving a maze
   - Certain mazes can be solved using the “right-hand” rule
   - Other mazes, e.g., with islands, require another approach
   - If you have “markers”, leave them at intersections, don’t explore the same place twice

What happens if you try to search the web, using links on pages to explore other links, using those links to ...
   - How many web pages are there?
   - What rules do web crawlers/webspiders follow?
     • Who enforces the rules?

Keep track of where you’ve been don’t go there again
   - Any problems with this approach?
Classic problem: N queens

- Can queens be placed on a chess board so that no queens attack each other?
  - Easily place two queens
  - What about 8 queens?
- Make the board NxN, this is the N queens problem
  - Place one queen/column
  - # different tries/column?
- Backtracking
  - Use “current” row in a col
  - If ok, try next col
  - If fail, back-up, next row
Backtracking idea with N queens

- **Try to place a queen in each column in turn**
  - Try first row in column $C$, if ok, move onto next column
  - If solved, great, otherwise try next row in column $C$, place queen, move onto the next column
    - Must unplace the placed queen to keep going

- **What happens when we start in a column, where to start?**
  - If we fail, move back to previous column (which remembers where it is/failed)
  - When starting in a column anew, start at beginning
    - When backing up, try next location, not beginning

- **Backtracking in general, record an attempt go forward**
  - If going forward fails, undo the record and backup
Basic ideas in backtracking search

- **We need to be able to enumerate all possible choices/moves**
  - We try these choices in order, committing to a choice
  - If the choice doesn’t pan out we must undo the choice
    - This is the backtracking step, choices must be undoable

- **Process is inherently recursive, so we need to know when the search finishes**
  - When all columns tried in N queens
  - When we have found the exit in a maze
  - When every possible moved tried in Tic-tac-toe or chess?
    - Is there a difference between these games?

- **Summary:** enumerate choices, try a choice, undo a choice, this is **brute force** search: try everything
N queens backtracking: nqueens.cpp

```cpp
bool Queens::SolveAtCol(int col)
// pre: queens placed at columns 0,1,...,col-1
// post: returns true if queen can be placed in column col
//       and N queen problem solved (N is square board size)
{
    int k; int rows = myBoard.numrows();
    if (col == rows) return true;
    for(k=0; k < rows; k++) {
        if (!NoQueensAttackingAt(k,col)) {
            myBoard[k][col] = true; // place a queen
            if (SolveAtCol(col+1)) {
                return true;
            }
            myBoard[k][col] = false; // unplace the queen
        }
    }
    return false;
}
```
Computer v. Human in Games

- Computers can explore a large search space of moves quickly
  - How many moves possible in chess, for example?

- Computers cannot explore every move (why) so must use heuristics
  - Rules of thumb about position, strategy, board evaluation
  - Try a move, undo it and try another, track the best move

- What do humans do well in these games? What about computers?
  - What about at Duke?
Backtracking, minimax, game search

- We’ll use tic-tac-toe to illustrate the idea, but it’s a silly game to show the power of the method
  - What games might be better? Problems?

- Minimax idea: two players, one maximizes score, the other minimizes score, search complete/partial game tree for best possible move
  - In tic-tac-toe we can search until the end-of-the game, but this isn’t possible in general, why not?
  - Use static board evaluation functions instead of searching all the way until the game ends

- Minimax leads to alpha-beta search, then to other rules and heuristics
Minimax for tic-tac-toe (see ttt.cpp)

- Players alternate, one might be computer, one human (or two computer players)

- Simple rules: win scores +10, loss scores -10, tie is zero
  - X maximizes, O minimizes

- Assume opponent plays smart
  - What happens otherwise?

- As game tree is explored is there redundant search?
  - What can we do about this?
The words above represent a simple substitution cypher
- Each letter mapped to one other letter, no inconsistencies
- Often used in cryptogram puzzles (newspaper, online, ...)
- How can we write a computer program to solve this?

Ideas for solving the problem? Benchmark/ballpark idea to accept (or not)

Problems on the horizon?
One possible solution in docrypto.cpp

- **Study this for an example of backtracking**
  - Similar to N queens: make move, recurse, undo as needed
  - What’s a move in this problem?

- **Illustrates a few C++ and OO concepts**
  - Static variables and functions: belong to class not object
  - Also called “class variables”, don’t need object to access
  - Must be careful when initializing static variables because order of initialization can be important

- **See WordSource object shared by all CryptoMap objects, how and when is the WordSource initialized?**
Heuristics

- A heuristic is a rule of thumb, doesn’t always work, isn’t guaranteed to work, but useful in many/most cases
  - Search problems that are “big” often can be approximated or solved with the right heuristics

- What heuristic is good for cryptograms?
  - Solve small words first
  - Solve large words first
  - Do something else?

- What other optimizations/improvements can we make?
  - See program, `cryptomap.cpp` and `docrypto.cpp`
**Towers of Hanoi**

- Move disks from "from" peg to "to" peg
- What is the recurrence relation in terms of numDisks?

```cpp
void Move(int from, int to, int aux, int numDisks) {
    // pre: numDisks on peg from,
    // post: numDisks moved to peg to
    
    if (numDisks == 1) {
        cout << from << " to " << to << endl;
    } else {
        Move(from, aux, to, numDisks-1);
        Move(from, to, aux, 1);
        Move(aux, to, from, numDisks-1);
    }
}
```