Balanced Search Trees

- **BST:** efficient lookup, insertion, deletion
  - Average case: $O(\log n)$ for all operations since find is $O(\log n)$ [complexity of insert after find is $O(1)$, why?]
  - Worst case is bad, what's big-Oh? What's the tree shape?
  - If we can guarantee $\log n$ in worst case, why might this be preferable to hashing? Properties of search tree?

- **Balanced Search trees**
  - Use rotations to maintain balance, different implementations rotate/rebalance at different times
  - AVL tree is conceptually simple, bookkeeping means coefficient for big-Oh is higher than other ideas
  - Red-black tree harder to code but good performance: basis for Java map classes and most C++ map classes
Balance trees we won't study

- **B-trees are used when data is both in memory and on disk**
  - File systems, really large data sets
  - Rebalancing guarantees good performance both asymptotically and in practice. Differences between cache, memory, disk are important

- **Splay trees rebalance during insertion and during search, nodes accessed often more closer to root**
  - Other nodes can move further from root, consequences?
    - Performance for some nodes gets better, for others ...
  - No guarantee running time for a single operation, but guaranteed good performance for a sequence of operations, this is good *amortized* cost (vector push_back)
Balanced trees we will study

- Both kinds have worst-case O(log n) time for tree operations
- AVL (Adel’son-Velskii and Landis), 1962
  - Nodes are “height-balanced”, subtree heights differ by 1
  - Rebalancing requires per-node bookkeeping of height
  - [http://www.seanet.com/users/arsen/avltree.html](http://www.seanet.com/users/arsen/avltree.html)

- Red-black tree uses same rotations, but can rebalance in one pass, contrast to AVL tree
  - In AVL case, insert, calculate balance factors, rebalance
  - In Red-black tree can rebalance on the way down, code is more complex, but doable
  - STL in C++ uses red-black tree for map and set classes
  - Standard java.util.TreeMap/TreeSet use red-black
Rotations and balanced trees

- **Height-balanced trees**
  - For every node, left and right subtree heights differ by at most 1
  - After insertion/deletion need to rebalance
  - Every operation leaves tree in a balanced state: *invariant property* of tree

- **Find deepest node that’s unbalanced**
  - On path from root to inserted/deleted node
  - Rebalance at this unbalanced point only

Are these trees height-balanced?
Rotation to rebalance

- When a node \( N \) is unbalanced, height differs by 2 (must be more than one)
  - Change \( N \rightarrow \text{left} \rightarrow \text{left} \)
    - \text{doLeft}
  - Change \( N \rightarrow \text{left} \rightarrow \text{right} \)
    - \text{doLeftRight}
  - Change \( N \rightarrow \text{right} \rightarrow \text{left} \)
    - \text{doRightLeft}
  - Change \( N \rightarrow \text{right} \rightarrow \text{right} \)
    - \text{doRight}

- First/last cases are symmetric
- Middle cases require two rotations
  - First of the two puts tree into \text{doLeft} or \text{doRight}

```c
Tree * doLeft(Tree * root)
{
    Tree * newRoot = root->left;
    root->left = newRoot->right;
    newRoot->right = root;
    return newRoot;
}
```
Rotation up close (doLeft)

- Why is this called doLeft?
  - N will no longer be root, new value in left->left subtree
  - Left child becomes new root

- Rotation isn’t “to the left”, but rather “brings left child up”
  - doLeftChildRotate?

```c
Tree * doLeft(Tree * root) {
    Tree * newRoot = root->left;
    root->left = newRoot->right;
    newRoot->right = root;
    return newRoot;
}
```
Rotation to rebalance

- Suppose we add a new node in right subtree of left child of root
  - Single rotation can’t fix
  - Need to rotate twice

- First stage is shown at bottom
  - Rotate blue node right
    - (its right child takes its place)
  - This is left child of unbalanced

```c
Tree * doRight(Tree * root) {
  Tree * newRoot = root->right;
  root->right = newRoot->left;
  newRoot->left = root;
  return newRoot;
}
```
Calculate where to rotate and what case, do the rotations

Tree * doRight(Tree * root)
{
    Tree * newRoot = root->right;
    root->right = newRoot->left;
    newRoot->left = root;
    return newRoot;
}

Tree * doLeft(Tree * root)
{
    Tree * newRoot = root->left;
    root->left = newRoot->right;
    newRoot->right = root;
    return newRoot;
}
AVL tree practice

- **Insert into AVL tree:**
  - 18 10 16 12 6 3 8 13 14
  - After adding 16: doLeftRight

- After 3, doLeft on 16
AVL practice: continued, and finished

- After adding 13, ok
- After adding 14, not ok
  - `doRight` at 12
Trie: efficient search of words/suffixes

- A trie (from retrieval, but pronounced “try”) supports
  - Insertion: a word into the trie (delete and look up)
  - These operations are $O(\text{size of string})$ regardless of how many strings are stored in the trie! Guaranteed!

- In some ways a trie is like a 128 (or 26 or alphabet-size) tree, one branch/edge for each character/letter
  - Node stores branches to other nodes
  - Node stores whether it ends the string from root to it

- Extremely useful in DNA/string processing
  - monkeys and typewriter simulation which is similar to some methods used in Natural Language understanding (n-gram methods)
Trie picture and code (see trie.cpp)

- To add string
  - Start at root, for each char create node as needed, go down tree, mark last node

- To find string
  - Start at root, follow links
    - If NULL, not found
  - Check word flag at end

- To print all nodes
  - Visit every node, build string as nodes traversed

- What about union and intersection?

  - Indicates word ends here
Boggle: Tries, backtracking, structure

Find words on 4x4 grid

- Adjacent letters:
  - ← → ↑ ↓ ↙ ↘ ↖ ↘
- No re-use in same word

Two approaches to find all words

- Try to form every word on board
  - Look up prefix as you go
    - Trie is useful for prefixes

- Look up every word in dictionary
  - For each word: on board?

- ZEAL and SMILES
void search(int row, int col, TrieNode * t, string soFar) {
    // pre: row, col are on board, soFar is valid prefix,
    //      string constructed on board during current search
    //      t represents the path in the trie of soFar
    if (!legal(row, col) || isVisited(row, col)) return;

    char ch = myBoard[row][col]; // check if still a prefix
    Node * child = t->links[ch]; // NOT a prefix, stop
    if (child == 0) return;       // still prefix, continue

    markVisited(row, col);        // don’t revisit
    string newPrefix = word + ch;
    if (child->isWord) cout << newPrefix << endl;

    doFind(row-1, col-1, child, newPrefix); // up-left
    doFind(row-1, col, child, newPrefix);    // straight up
    doFind(row-1, col+1, child, newPrefix);  // still 5 more calls
    unVisit(row, col);                   // now ok to revisit
}
bool wordFound(const string& s, const Point& p)
// pre: s is suffix of word searched for, prefix so far
//      is found and last letter of found prefix at p(row,col)
{
    if (s.length() == 0) return true;  // no more suffix, done

tvector<Point> points = myPointsFor(s[0]);

for(int k=0; k < points.size(); k++) {
    Point nextP = points[k];
    if (IsAdjacent(p,nextP) && ! isVisited(nextP)) {
        markVisited(nextP);     // don’t visit again
        if (wordFound(s.substr(1,s.length()-1),nextP)) {
            return true;
        }
        unVisit(nextP);         // ok to visit again
    }
    unVisit(nextP);            // ok to visit again
}
return false;  // tried to find s, failed in all attempts
Nancy Leveson: Software Safety

Founded the field
- Mathematical and engineering aspects
  - Air traffic control
  - Microsoft word

"C++ is not state-of-the-art, it's only state-of-the-practice, which in recent years has been going backwards"

- Software and steam engines: once extremely dangerous?
- THERAC 25: Radiation machine that killed many people