Minimum Cut

deterministic algorithms

- Max-flow
- Gabow

Min-cut implementation

- data structure for contractions
- alternative view—permutations.
- deterministic leaf algo

Recursion:

\[ p_{k+1} = p_k - \frac{1}{4}p_k^2 \]
\[ q_k = \frac{4}{p_k} + 1 \]
\[ q_{k+1} = q_k + 1 + \frac{1}{q_k} \]

MST

Background

- kruskal
- boruvka
- verification

sampling theorem:

- Heavy edges

- pick F with probability p
- get \( n/p \) F-heavy edges

Recursive algorithm without boruvka:

\[ T(m, n) = T(m/2, n) + O(m) + T(2n, n) = O(m + n \log n) \]

(sloppy on expectation)

Recursive algorithm with 3 boruvka steps:

\[ T(m, n) = T(m/2, n/8) + c_1(m + n) + T(n/4, n/8) \]
\[ \leq c(m/2 + n/8) + c_1(m + n) + c(n/4 + n/8) \]
\[ = (c/2 + c_1)m + (c/8 + c_1 + c/4 + c/8)n \]
\[ = (c/2 + c_1)(m + n) \]

so set \( c = 2c_1 \)

Notes:
• Chazelle $m \log \alpha(m, n)$ via relaxed heap
• open questions.

Samples for Cuts
• cut counting
• Reliability
• Gabow’s algorithm
• Sampling