1. Let $X$ be a non-negative random variable such that $E[X]$ is well defined. Using this, prove, for all $t > 0$, Markov’s Inequality:

\[
\Pr[X \geq t] \leq \frac{E[X]}{t}
\]

2. Consider the Balls and Bins problem with $m$ bins and $n$ balls, with $n \geq m$. Assuming that when balls are thrown into the bins, they’re placed uniformly. Now compute (in terms of $m$ and $n$) the probability that:

a) There is a bin with no balls in it.

b) At least $k$ of the bins are empty.

c) There is a bin with less than $n/m$ balls in it.

Now consider if the probability is not uniformly distributed, but rather follows a geometric distribution; that is, the probability of landing in bucket 1 is $p$, bucket 2 is $(1-p)p$, bucket 3 is $(1-p)^2p$, and so on, until the bucket $m$ is the remainder of the probability $(1 - \sum_{i=0}^{m-1}(1-p)^i p)$.

d) What is the probability that bucket $i$ is empty?

e) What is the probability that no bucket is empty?

f) Compute the expected number of balls in each bucket.

3. Show how we can use two stacks to implement a Queue such that the amortized cost of each Enqueue and Dequeue is $O(1)$.

4. Show how we can implement a dynamic set that efficiently supports three operations: Enqueue, Dequeue and Min. How can we implement this in:

a) Enqueue and Dequeue in $O(1)$ time, Min in $O(n)$ time.

b) Enqueue and Dequeue in $O(\log n)$ time, Min in $O(1)$ time.

c) All three operations in amortized $O(1)$ time.

Extra Credit Suppose you’re given a list of $n$ integers, some negative, some positive. We want to find any three distinct integers from the list so that $a + b + c = 0$. How can we do this in $O(n^3)$ time? Describe how we can reduce this to $O(n^2)$. Can we do better? This problem is not easy, finish the actual homework problems before trying to do it.