Indexing: Part II

CPS 216
Advanced Database Systems

Announcements (February 8)
- Homework #1 due today
- No class this Thursday (February 10)
- Reading assignments this week
  - Generalized search trees (due next Tuesday)
  - "The" Google paper (due next Thursday)

R-trees

- B-tree: balanced hierarchy of 1-d ranges
- R-tree: balanced hierarchy of n-d ranges

R-tree lookup

- Which ranges contain me?
- Problem: search may go down many paths
  - Because regions may overlap
  - No performance guarantee like B-tree

R-tree insertion

Insert $R_j$ into R-tree
- Start from the root
- Pick a region containing $R_j$ and follow the child pointer
  - If none contains $R_j$, pick one and grow it to contain $R_j$
  - Pick the one that requires the least enlargement (why?)

R-tree insertion: split

- If a node is too full, split
- Try to minimize the total area of bounding boxes
  - Exhaustive: try all possible splits
  - Quadratic: "seed" with the most wasteful pair; iteratively assign regions with strongest "preference"
  - Linear: "seed" with distant regions; iteratively assign others as Quadratic
R-tree insertion: split (cont’d)

- Split could propagate all the way up to the root (not shown in this example)

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R*-tree

- Always tries to minimize the area of bounding boxes
- Quadratic splitting algorithm encourages small seeds and possibly long and narrow bounding boxes
- R*-tree (Beckmann et al., *SIGMOD* 1990)
  - Consider other criteria, e.g.
    - Minimize overlap between bounding boxes
    - Minimize the margin (perimeter length) of a bounding box
  - Forced reinserts
    - When a node overflows, reinsert "outer" entries
    - They may be picked up by other nodes, thus saving a split

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R+-tree

- Problem with R-tree
  - Regions may overlap
  - Search may go down many paths
- R+-tree (Sellis et al., *VLDB* 1987)
  - Regions in non-leaf nodes do not overlap
  - Search only goes down one path
  - Duplicate items in leaves
  - But an insertion must now go down many paths!
    - R must be inserted into all R+-tree leaves whose bounding boxes overlap with R
  - A bigger tree

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Indexing user-defined data types

- Specialized indexes (ABCDEFG trees…)
  - Redundant code: most trees are very similar
  - Concurrency control and recovery especially tricky to get right
- Extensible B-trees and R-trees
  - Examples: B-trees in Berkeley DB, B- and R-trees in Informix
  - User-defined `compare()` function
- GiST (Generalized Search Trees)
  - General (covers B-trees, R-trees, etc.)
  - Easy to extend
  - Built-in concurrency control and recovery

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Structure of GiST

Balanced tree of \( \langle p, ptr \rangle \) pairs

- \( p \) is a key predicate that holds for all objects found below \( ptr \)
- Every node has between \( kM \) and \( M \) index entries…
  - \( k \) must be no more than \( \frac{1}{2} \) (why?)
- Except root, which only needs at least two children
- All leaves are on the same level

- User only needs to define what key predicates are
Defining key predicates

- **boolean** Consistent(entry entry, predicate query)
  - Return true if an object satisfying query might be found under entry
- **predicate** Union(set entry > entries)
  - Return a predicate that holds for all objects found under entries
- **real** Penalty(entry entry 1, entry entry 2)
  - Return a penalty for inserting entry 2 into the subtree rooted at entry 1
- **(set entry >, set entry >)** PickSplit(set entry > entries)
  - Given M+1 entries, split it into two sets, each of size at least kM

Index operations

- **Search**
  - Just follow pointer whenever Consistent() is true
- **Insert**
  - Descent tree along least increase in Penalty()
  - If there is room in leaf, insert there; otherwise split according to PickSplit()
  - Propagate changes up using Union()
- **Delete**
  - Search for entry and delete it
  - Propagate changes up using Union()
  - On underflow
    - If keys are ordered, can borrow/coalesce in B-tree style
    - Otherwise, reinsert stuff in the node and delete the node

GiST over R (B+-tree)

- Logically, keys represent ranges \([x, y]\)
- Query: find keys that overlap with \([a, b]\)
- Consistent(entry, \([a, b]\)): say entry has key \([x, y]\)
  - \(x < b\) and \(y > a\), i.e., overlap
- Union(entries): say entries = \([x, y]\)
  - \([\min(x), \max(y)]\)
- Penalty(entry 1, entry 2): say they have keys \([x_1, y_1]\) and \([x_2, y_2]\)
  - \(\max(y_1 - y_2, 0) + \max(x_2 - x_1, 0)\), except boundary cases
- PickSplit(entries)
  - Sort entries and split evenly
  - Plus a special Compare(entry, entry) for ordered keys

Key compression

- Without compression, GiST would need to store a range instead of a single key value in order to support B+-tree
- Two extra methods: Compress/Decompress
- For B+-tree
  - Compress(entry): say entry has key \([x, y]\)
    - \(x\), assuming next entry starts with \(y\), except boundary cases
  - Decompress\((x, ptr)\)
    - \([x, y]\), assuming next entry starts with \(y\), except boundary cases
  - This compression is lossless: \(Decompress(Compress(e)) = e\)

GiST over R² (R-tree)

- Logically, keys represent bounding boxes
- Query: find stuff that overlaps with a given box
  - Abusing notation a bit below...
- Consistent(key_box, query_box)
  - key_box overlaps with query_box
- Union(box)
  - Minimum bounding box of boxes
- Penalty(box 1, box 2)
  - Area of Union(box 1, box 2) – area of box
- PickSplit(box)
  - R-tree algorithms (e.g., minimize total area of bounding boxes)
- Compare(box, box)?

GiST over P(Z) (RD-tree)

- Logically, keys represent sets
- Queries: find all sets that intersect with a given set
- Consistent(key_set, query_set)
  - key_set intersects with query_set
- Union(set)
  - Union of sets
- Penalty(set 1, set 2)
  - \(|\text{Union(set 1, set 2)| - |set 1|}
- PickSplit(set)
  - Much like R-tree (e.g., minimize total cardinality)
- Compare(set, set)?
- Compress/Decompress: bloom filters, rangesets, etc.
  - Decompress(Compress(set)) set
  - Lossy: Decompress(Compress(set)) \(\supseteq set\)
Next

- Hash-based indexing
- Text indexing