Combining Turing Machines

We will define notation that will make it easier to look at more complicated Turing machines.

1. Given Turing Machines $M_1$ and $M_2$
   
   Notation for
   - Run $M_1$
   - Run $M_2$

2. Given Turing Machines $M_1$ and $M_2$
   
   Notation for
   - Run $M_1$
   - If $x$ is current symbol
     - then Run $M_2$
3. Given Turing Machines M1, M2, and M3

Notation for
- Run M1
- If x is current symbol
  - then Run M2
  - else Run M3

More Notation for Simplifying Turing Machines

Suppose Γ=\{a,b,c,B\}

- z is any symbol in Γ
- x is a specific symbol from Γ

1. s - start
2. R - move right
3. L - move left

4. x - write x (and don’t move)

5. Rₐ - move right until you see an a

6. Lₐ - move left until you see an a

7. Rₐₐ - move right until you see anything that is not an a

8. Lₐₐ - move left until you see anything that is not an a

9. h - halt in a final state

10. \( a, b \to w \)

   If the current symbol is a or b, let w represent the current symbol.
Example
Assume input string $w \in \Sigma^+$, $\Sigma = \{a, b\}$.

If $|w|$ is odd, then write a $b$ at the end of the string. The tape head should finish pointing at the leftmost symbol of $w$.

input: bab, output: babb

input: ba, output: ba

What is the running time?
Example
Assume input string \( w \in \Sigma^+ \), \( \Sigma = \{a, b\} \), \( |w| > 0 \)

For each \( a \) in the string, append a \( b \) to the end of the string.

input: \( abbabb \), output: \( abbabbb \)

The tape head should finish pointing at the leftmost symbol of \( w \).

Turing’s Thesis Any computation that can be carried out by a mechanical means can be performed by a TM.

Definition: An algorithm for a function \( f:D \rightarrow R \) is a TM \( M \), which given input \( d \in D \), halts with answer \( f(d) \in R \).

Example: \( f(x + y) = x + y \), \( x \) and \( y \) unary numbers.

<table>
<thead>
<tr>
<th>start with:</th>
<th>111+1111</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>↑</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>end with:</th>
<th>1111111</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>↑</td>
</tr>
</tbody>
</table>
Example: Copy a String, \( f(w) = w0w \), \( w \in \Sigma^* \), \( \Sigma = \{a, b, c\} \)

Denoted by \( C \)

\[
\begin{array}{c}
\text{start with: } & \text{abac} \\
\uparrow \\
\text{end with: } & \text{abac0abac} \\
\uparrow \\
\end{array}
\]

Algorithm:

- Write a 0 at end of string
- For each symbol in string
  - make a copy of the symbol

\[
\begin{array}{c}
s \ R \ 0 \quad L \quad R \\
B \quad B \\
\downarrow \\
0 \\
\end{array} \quad \left\{ a,b,c \right\} \quad \begin{array}{c}
w \\
B \quad R \quad w \quad L \quad w \\
B \quad B \\
\end{array}
\]
**Example:** Shift the string that is to the left of the tape head to the right, denoted by $S_R$ (shift right)

Below, “ba” is to the left of the tape head, so shift “ba” to the right.

```
   start with:     aaBbabca
    ↑

   end with:      aaBBbaca
    ↑
```

Algorithm:

- remember symbol to the right and erase it
- for each symbol to the left do
  - shift the symbol one cell to the right
- replace first symbol erased
- move tape head to appropriate position
Example: Shift the string that is to the right of tape head to the left, denote by $S_L$ (shift left)

start with:  babcaBba
            ↑
end with:    bacaBBba
            ↑

(similar to $S_R$)
Example: Add unary numbers

This time use shift.

Example: Multiply two unary numbers, $f(x \cdot y) = x \cdot y$, $x$ and $y$ unary numbers. Assume $x, y > 0$.

\[
\text{start with: } \quad 1111 \times 11 \\
\quad \uparrow \\
\text{end with: } \quad 11111111 \\
\quad \uparrow 
\]