Section: Turing Machines - Building Blocks

1. Given Turing Machines M1 and M2

Notation for

- Run M1
- Run M2

$M_2 M_1$

$H'S'S H$

$M_1 M_2$

$z;z,R ~ z;z,L$

$z$ represents any symbol in

$M_2 M_1$

$H'S'S H$

$M_1 M_2$

$z;z,R ~ z;z,L$

$z$ represents any symbol in
2. Given Turing Machines $M_1$ and $M_2$

$M_1$

$M_2$

$x$ represents any symbol in $x \in \{x, x; R, z; z; L\}$.
3. Given Turing Machines $M_1$, $M_2$, and $M_3$

$x$ is an element of
$y$ is any element except $x$ from
$z$ is any element from
More Notation for Simplifying Turing Machines

Suppose $\Gamma=\{a, b, c, B\}$

$z$ is any symbol in $\Gamma$

$x$ is a specific symbol from $\Gamma$

1. $s$ - start
2. $R$ - move right

3. $L$ - move left

4. $x$ - write $x$ (and don’t move)

5. $R_a$ - move right until you see an $a$
6. $L_a$ - move left until you see an $a$

7. $R_{\neg a}$ - move right until you see anything that is not an $a$

8. $L_{\neg a}$ - move left until you see anything that is not an $a$

9. $h$ - halt in a final state

10. $a, b \rightarrow w$

   If the current symbol is $a$ or $b$, let $w$ represent the current symbol.
Example
Assume input string $w \in \Sigma^+$, $\Sigma = \{a, b\}$. If $|w|$ is odd, then write a $b$ at the end of the string. The tape head should finish pointing at the leftmost symbol of $w$.

input: bab, output: babb
input: ba, output: ba

What is the running time?
Example

Assume input string $w \in \Sigma^+, \Sigma = \{a, b\}$, $|w| > 0$

For each $a$ in the string, append a $b$ to the end of the string.

input: $abbabb$, output: $abbabbbb$

The tape head should finish pointing at the leftmost symbol of $w$. 
Turing’s Thesis Any computation that can be carried out by a mechanical means can be performed by a TM.

Definition: An algorithm for a function $f: D \to R$ is a TM $M$, which given input $d \in D$, halts with answer $f(d) \in R$.

Example: $f(x + y) = x + y$, $x$ and $y$ unary numbers.

\[\text{start with: } 111 + 1111 \quad \uparrow \]

\[\text{end with: } 1111111 \quad \uparrow \]
Example: Copy a String, \( f(w) = w0w \), \( w \in \Sigma^* \), \( \Sigma = \{a, b, c\} \)

Denoted by \( C \)

\[
\begin{align*}
\text{start with:} & \quad \text{abac} \\
\uparrow \\
\text{end with:} & \quad \text{abac0abac}
\end{align*}
\]

Algorithm:

- Write a 0 at end of string
- For each symbol in string
  - make a copy of the symbol
Example: Shift the string that is to the left of the tape head to the right, denoted by $S_R$ (shift right)

Below, “ba” is to the left of the tape head, so shift “ba” to the right.

start with: $aaBbabca$

\[\uparrow\]

end with: $aaBBbaca$

\[\uparrow\]
Algorithm:

• remember symbol to the right and erase it

• for each symbol to the left do
  – shift the symbol one cell to the right

• replace first symbol erased

• move tape head to appropriate position
Example: Shift the string that is to the right of tape head to the left, denote by $S_L$ (shift left)

\[
\begin{align*}
\text{start with:} & \quad \text{babcaBba} \\
& \uparrow \\
\text{end with:} & \quad \text{bacaBBba} \\
& \uparrow \\
\end{align*}
\]

(similar to $S_R$)
Example: Add unary numbers
This time use shift.

Example: Multiply two unary numbers, \( f(x \times y) = x \times y \), x and y unary numbers. Assume \( x, y > 0 \).

\[
\begin{align*}
\text{start with:} & \quad 1111 \times 11 \\
& \uparrow \\
\text{end with:} & \quad 11111111 \\
& \uparrow 
\end{align*}
\]