Review

Regular Languages

- FA, RG, RE
- recognize

Context Free Languages

- PDA, CFG
- recognize

DFA:

Turing Machine:
Turing Machine (TM)

- invented by Alan M. Turing (1936)
- computational model to study algorithms

Definition of TM

- Storage
  - tape
- actions
  - write symbol
  - read symbol
  - move left (L) or right (R)
- computation
  - initial configuration
    - start state
    - tape head on leftmost tape square
    - input string followed by blanks
  - processing computation
    - move tape head left or right
    - read from and write to tape
  - computation halts
    - final state

Formal Definition of TM

A TM $M$ is defined by $M=(Q,\Sigma,\Gamma,\delta,q_0,B,F)$ where

- $Q$ is finite set of states
- $\Sigma$ is input alphabet
- $\Gamma$ is tape alphabet
- $B \in \Gamma$ is blank
- $q_0$ is start state
- $F$ is set of final states
- $\delta$ is transition function
  
  $\delta(q,a) = (p,b,R)$ means “if in state $q$ with the tape head pointing to an ’a’, then move into state $p$, write a ’b’ on the tape and move to the right”.


TM as Language recognizer

Definition: Configuration is denoted by \( \vdash \).

If \( \delta(q, a) = (p, b, R) \) then a move is denoted

\[
\text{abaqabba} \vdash \text{ababpbba}
\]

Definition: Let \( M \) be a TM, \( M=(Q, \Sigma, \Gamma, \delta, q_0, B, F) \). \( L(M) = \{ w \in \Sigma^* | q_0 w \vdash x_1 q_f x_2 \text{ for some } q_f \in F, x_1, x_2 \in \Gamma^* \} \)

TM as language acceptor

\( M \) is a TM, \( w \) is in \( \Sigma^* \),

- if \( w \in L(M) \) then \( M \) halts in final state
- if \( w \notin L(M) \) then either
  - \( M \) halts in non-final state
  - \( M \) doesn’t halt

Example

\( \Sigma = \{a, b\} \)

Replace every second 'a' by a 'b' if string is even length.

- Algorithm
Example:

$L = \{ a^n b^n c^n | n \geq 1 \}$

Is the following TM Correct?

**TM as a transducer**

TM can implement a function: $f(w) = w'$

```
start with:  \[ w \]

end with: \[ w' \]
```
**Definition:** A function with domain \( D \) is *Turing-computable* or *computable* if there exists TM \( M=(Q,\Sigma,\Gamma,\delta,q_0,B,F) \) such that

\[
q_0w \vdash^* q_f f(w)
\]

\( q_f \in F \), for all \( w \in D \).

**Example:**

\( f(x) = 2x \)

\( x \) is a unary number

Start with: 111

\[ \uparrow \]

End with: 111111

\[ \uparrow \]

Is the following TM correct?
Example:

$L = \{ww \mid w \in \Sigma^+\}, \Sigma = \{a, b\}$