1 Greedy Algorithm (20 points)

Consider the following single machine preemptive scheduling problem. We are given a set of \( n \) jobs \( 1, 2, \ldots, n \), with each job having a processing length \( p_1, p_2, \ldots, p_n \) and release date \( r_1, r_2, \ldots, r_n \). The job \( i \) can be scheduled only after its release date \( r_i \). Completion time of job \( i \), denoted by \( C_i \), is the earliest time \( t \) at which job is completely processed by the machine. Design an \( O(n \log n) \)-time algorithm to find a schedule that minimizes the total completion time, i.e., minimizes \( \sum_i C_i \).

2 Dynamic Programming, DPV 6.1 (20 points)

A contiguous subsequence of a list \( S \) is a subsequence made up of consecutive elements of \( S \). For instance, if \( S \) is 5, 15, -30, 10, -5, 40, 10; then 15, 30, 10 is a contiguous subsequence but 5, 15, 40 is not. Give a linear-time algorithm for the following task:

**Input:** A list of numbers, \( a_1, a_2, \ldots, a_n \).

**Output:** The contiguous subsequence of maximum sum (a subsequence of length zero has sum zero).

**Hint:** For each \( j \in \{1, 2, \ldots, n\} \) consider contiguous subsequences ending exactly at position \( j \).

3 Dynamic Programming, DPV 6.3 (20 points)

Yuckdonald’s is considering opening a series of restaurants along Quaint Valley Highway (QVH). The \( n \) possible locations are along a straight line, and the distances of these locations from the start of QVH are, in miles and in increasing order, \( m_1, m_2, \ldots, m_n \). The constraints are as follows:

- At each location, Yuckdonald’s may open at most one restaurant. The expected profit from opening a restaurant at location \( i \) is \( p_i \), where \( p_i > 0 \) and \( i = 1, 2, \ldots, n \).
- Any two restaurants should be at least \( k \) miles apart, where \( k \) is a positive integer.

Give an efficient algorithm to compute the maximum expected total profit subject to the given constraints.

4 More Dynamic Programming, DPV 6.7 (20 points)

A subsequence is palindromic if it is the same whether read left to right or right to left. For instance, the sequence \( A, C, G, T, G, T, C, A, A, A, T, C, G \) has many palindromic subsequences, including \( A, C, G, C, A \) and \( A, A, A, A \) (on the other hand, the subsequence \( A, C, T \) is not palindromic). Devise an \( O(n^2) \)-time algorithm that takes a sequence \( x[1 \ldots n] \) and returns the (length of the) longest palindromic subsequence.
Let $P$ be a convex polygon with $n$ vertices. A line segment connecting any two vertices $u$ and $v$ of $P$ lies completely inside $P$; the weight of $uv$, denoted by $w_{uv}$, is the Euclidean distance between the vertices $u$ and $v$. A triangulation of $P$ polygon is a decomposition of the polygon into $n - 2$ triangles. (See figures below). Notice that there are several ways of triangulating $P$. We define the weight of a triangulation of $P$ to be the sum of the weights of its chords. Describe an $O(n^3)$-time algorithm to compute a minimum-weight triangulation of $P$.

Figure 1: Two triangulations of a convex polygon.