Due Date: Thursday, Feb 16, 2012

1  Sorting [DPV 2.17] (20 points)
Given a sorted array of distinct integers $A[1...n]$, describe an $O(\log n)$-time algorithm to
determine whether there is an index $i$ such that $A[i] = i$.

2  Merging sorted arrays [DPV 2.22] (20 points)
Given two sorted lists of size $m$ and $n$ and an integer $1 \leq k \leq m + n$, describe an $O(\log m +$
log n)$ time algorithm for computing the $k$th smallest element in the union of two lists.

3  Finding the majority element [DPV 2.23] (20 points)
An array $A[1...n]$ is said to have a majority element if more than half of its entries are same.
Given an array, task is to design an efficient algorithm to tell whether array has a majority
element, and if so, find the element. The elements of the array are not necessarily from
some ordered domain, so only allowed operation is query of the form $A[i] = A[j]$.

   • Show how to solve this problem in $O(n \log n)$ time.
     (Hint: Divide the array into two smaller arrays. Does knowing the majority element
     of them help to figure out the majority element of $A$?)

   • Give a linear time algorithm for the same problem.
     (Hint: Here is another approach. Pair up the elements of array to get $\frac{n}{2}$ pairs. In
each pair, if elements are different discard both of them. If they are same, then keep
one of them. Show that after this procedure, there are at most $\frac{n}{2}$ elements left and
they have a majority element)

4  Bipartite graphs [DPV 3.7] (20 points)
A bipartite graph is a graph $G = (V, E)$ whose vertices can be partitioned into two sets
$(V = V_1 \cup V_2)$ and $V_1 \cap V_2 = \emptyset$ such that there are no edges between vertices in the same
set.

   • Give a linear-time algorithm to determine whether an undirected graph is bipartite.

   • Prove that an undirected graph is bipartite if and only if it contains no cycles of odd
length.

5  Finding Cycles [DPV 3.11] (20 points)
Design a linear-time algorithm which, given an undirected graph $G$ and an particular edge
e in it, determines whether $G$ has a cycle containing $e$. 