Experimenting with Grammars to Generate L-Systems – in JFLAP
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L-Systems

- Grammatical systems introduced by Lyndenmayer
- Model biological systems and create fractals
- Similar to Chomsky grammars, except all variables are replaced in each step, not just one!
- Successive strings are interpreted as strings of render commands and displayed graphically
Parts of an L-System  
(a type of grammar)

- Defined over an alphabet
- Three parts
  - Axiom (starting place)
  - Replacement rules (replaces all variables at once)
  - Geometric rules (for drawing)
    - \texttt{g} means move forward one unit with pen down
    - \texttt{f} means move forward one unit with pen up
    - \texttt{+} means turn right by the default angle
    - \texttt{-} means turn left by the default angle
L-System

An L-system is composed of three parts $(\Sigma, h, w)$

- $\Sigma$ finite alphabet set of symbols
- $h$ rewriting rules each symbol is replaced by string of symbols
- $w$ axiom starting point

$h$ is finite substitutions, $h: \Sigma \rightarrow \Sigma^*$. 
h(w)

h(w) is computed by replacing every symbol in w that has a rewrite rule by that rule.

A language L of an L-system is the word sequence generated by

\[
\begin{align*}
\bullet h^0(w) &= w \\
\bullet h^1(w) &= h(w) \\
\bullet h^2(w) &= h(h(w)) \\
\bullet \ldots \\
L &= \{h^i(w) \mid i \geq 0\}
\end{align*}
\]
NOTE: If $h(a) = bb$ we will write this as a rule

$$a \rightarrow bb$$
Example:

\[ \Sigma \] alphabet: \( \{a, b\} \)

h rules: \( a \rightarrow aa \)

\( b \rightarrow ab \)

w axiom: \( ab \)

What is the language \( L \) of strings represented by this \( L \)-system?

\( L = \)
Drawing a picture of an L-system

Defining an L-system: (3 parts in this order)

- Axiom definition: This must be the first line of the file
- Production rules: Defines the replacement rules.
- Geometric rules: Defines colors, widths, etc.
Graphically represent

Symbols for drawing and moving:

- $g$: draw a line one step in the current direction
- $f$: move forward one step in the current direction
Example: example1

axiom X

X \rightarrow g \, f \, g \, X

distance 15
lineWidth 5
color black

L =

What does this draw?
Geometric rules

- +  change direction to the right
- -  change direction to the left
- %  change direction 180 degrees
- ~  decrement the width of the next lines
- [  save in stack current state info
- ]  recover from stack state info
- {  start filled in polygon
- }  end filled in polygon
Example – lsys-samp1

- Axiom
- Replacement Rules
- Geometric Rules

NOTE: Must use spaces as separator between symbols
Example – lsys-samp1 (cont)

• Derivation of strings

\[
\begin{align*}
X & \quad gggX + Y \\
ggggggX + Y + g \\
ggggggX + Y + g + g \\
ggggggX + Y + g + g + g \\
\end{align*}
\]

Note: replace both X and Y each time.
Example – lsys-samp2
Example – lsys-samp2 (cont)

\[ g[\sim+Yg]gX \]

\[ g[\sim++Yg]gg[\sim+Yg]gX \]

\[ g[\sim+++Yg]gg[\sim++Yg]gg[\sim+Yg]gX \]

\[ \ldots \]
Example - tree

```
Axiom: R ~ ## B

B → [~ ## TL - B ++ B]
L → [{ - g ++ g % -- g }]
R → ! @@ R
T → T g
```

<table>
<thead>
<tr>
<th>Name</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>color</td>
<td>brown</td>
</tr>
<tr>
<td>polygonColor</td>
<td>forestGreen</td>
</tr>
</tbody>
</table>
Example – tree rendered
Stochastic Tree

- Add a rule $T \rightarrow T$
- Now there is a choice for $T$, draw a line or don’t
Same Stochastic L-System

- Rendered 3 times, each at 8\textsuperscript{th} derivation
• JFLAP is available for free:

  www.jflap.org

• Duke School of Environment uses L-systems to model pine needles in Duke Forest
Classwork 5 - Exercise 1

• Write an L-system for the picture below.
• Symbols needed are: g, + and one variable
• Distance of the line is 100, rendering at 1 draws the first line, each additional render draws another line.
Exercise 2

• Write an L-system for the picture below.
• Symbols may need: g, %, +
• Distance set to 15, angle set to 45, side of square is length 30, first diagonal line is 60
• 1st, 2nd and 6th renderings shown
Exercise 3

- Write an L-system for the picture below.
- Symbols may need: g, +, -, [ ]
- Angle set to 90, distance set to 15
- Shows 1st, 2nd and 3rd renderings