Combining Turing Machines

We will define notation that will make it easier to look at more complicated Turing machines.

1. Given Turing Machines M1 and M2
   Notation for
   • Run M1
   • Run M2

2. Given Turing Machines M1 and M2
   Notation for
   • Run M1
   • If x is current symbol
     – then Run M2
3. Given Turing Machines M1, M2, and M3

Notation for
- Run M1
- If x is current symbol
  - then Run M2
  - else Run M3

More Notation for Simplifying Turing Machines

Suppose $\Gamma = \{a, b, c, B\}$

- z is any symbol in $\Gamma$
- x is a specific symbol from $\Gamma$

1. s - start
2. R - move right
3. L - move left

4. x - write x (and don’t move)

5. R_a - move right until you see an \( a \)

6. L_a - move left until you see an \( a \)

7. R_{\sim a} - move right until you see anything that is not an \( a \)

8. L_{\sim a} - move left until you see anything that is not an \( a \)

9. h - halt in a final state

10. \( \frac{a,b}{w} \rightarrow \)

If the current symbol is a or b, let \( w \) represent the current symbol.
Example

Assume input string $w \in \Sigma^+$, $\Sigma = \{a, b\}$.

If $|w|$ is odd, then write a $b$ at the end of the string. The tape head should finish pointing at the leftmost symbol of $w$.

input: bab, output: babb

input: ba, output: ba

What is the running time?
Example

Assume input string $w \in \Sigma^+, \Sigma = \{a, b\}, |w| > 0$

For each $a$ in the string, append a $b$ to the end of the string.

input: $abbabb$, output: $abbabbbb$

The tape head should finish pointing at the leftmost symbol of $w$.

Turing’s Thesis Any computation that can be carried out by a mechanical means can be performed by a TM.

Definition: An algorithm for a function $f:D \rightarrow R$ is a TM $M$, which given input $d \in D$, halts with answer $f(d) \in R$.

Example: $f(x + y) = x + y$, $x$ and $y$ unary numbers.

start with: 111+1111
        ↑

end with: 1111111
        ↑
**Example:** Copy a String, \( f(w) = w0w, \ w \in \Sigma^*, \ \Sigma = \{a, b, c\} \)

Denoted by \( C \)

- start with: \( abac \)
  - \( \uparrow \)

- end with: \( abac0abac \)
  - \( \uparrow \)

**Algorithm:**

- Write a 0 at end of string
- For each symbol in string
  - make a copy of the symbol
**Example:** Shift the string that is to the left of the tape head to the right, denoted by $S_R$ (shift right)

Below, “ba” is to the left of the tape head, so shift “ba” to the right.

\[
\begin{align*}
\text{start with:} & \quad \text{aaBbabca} \\
\text{end with:} & \quad \text{aaBBbaca}
\end{align*}
\]

Algorithm:

- remember symbol to the right and erase it
- for each symbol to the left do
  - shift the symbol one cell to the right
- replace first symbol erased
- move tape head to appropriate position
**Example:** Shift the string that is to the right of tape head to the left,
denote by $S_L$ (shift left)

\[
\text{start with: } \quad \text{babcaBba}
\]
\[
\uparrow
\]
\[
\text{end with: } \quad \text{bacaBBba}
\]
\[
\uparrow
\]

(similar to $S_R$)

\[
\text{s} \quad \text{L} \quad a,b,c,B \quad \text{v} \quad 0
\]
\[
\text{R} \quad \text{B} \quad \text{R} \quad \text{w} \quad \text{B} \quad \text{L} \quad \text{w} \quad \text{R}
\]
\[
\text{B}
\]
\[
\text{L} \quad 0 \quad \text{v} \quad \text{R} \quad \text{h}
\]
Example: Add unary numbers

This time use shift.

Example: Multiply two unary numbers, \( f(x \times y) = x \times y \), \( x \) and \( y \) unary numbers. Assume \( x, y > 0 \).

start with: \hspace{1cm} 1111 \times 11 \hspace{1cm} \uparrow \\
end with: \hspace{1cm} 1111111 \hspace{1cm} \uparrow