1. Given Turing Machines $M_1$ and $M_2$

Notation for

- Run $M_1$
- Run $M_2$

$M_1 \rightarrow M_2$

$z$ represents any symbol in $M_2$.
2. Given Turing Machines M1 and M2

M1

\[ S \quad H \]

\[ \rightarrow \]

M2

\[ S' \quad H' \]

\[ \rightarrow \]

\[ \rightarrow M1 \xrightarrow{x} M2 \]

\[ S \quad H \xrightarrow{x;R,H} z;L \quad S' \quad H' \]

z represents any symbol in x
x is an element of
3. Given Turing Machines M1, M2, and M3

M1

M2

M3

x is an element of
y is any element except x from
z is any element from
More Notation for Simplifying Turing Machines

Suppose $\Gamma = \{a, b, c, B\}$

$z$ is any symbol in $\Gamma$

$x$ is a specific symbol from $\Gamma$

1. $s$ - start
2. $R$ - move right

3. $L$ - move left

4. $x$ - write $x$ (and don’t move)

5. $R_a$ - move right until you see an $a$
6. \( L_a \) - move left until you see an \( a \)

7. \( R_{\neg a} \) - move right until you see anything that is not an \( a \)

8. \( L_{\neg a} \) - move left until you see anything that is not an \( a \)

9. \( h \) - halt in a final state

10. \( \{a,b\} \rightarrow w \rightarrow \)

   If the current symbol is \( a \) or \( b \), let \( w \) represent the current symbol.
Example

Assume input string \( w \in \Sigma^+ \), \( \Sigma = \{a, b\} \).
If \(|w|\) is odd, then write a \( b \) at the end of the string. The tape head should finish pointing at the leftmost symbol of \( w \).

input: bab, output: babb
input: ba, output: ba

What is the running time?
Example

Assume input string \( w \in \Sigma^+, \Sigma = \{a, b\}, |w| > 0 \)

For each \( a \) in the string, append a \( b \) to the end of the string.

**input:** \( abbabb \), **output:** \( abbabbbb \)

The tape head should finish pointing at the leftmost symbol of \( w \).
Turing’s Thesis Any computation that can be carried out by a mechanical means can be performed by a TM.

Definition: An algorithm for a function $f: D \rightarrow R$ is a TM $M$, which given input $d \in D$, halts with answer $f(d) \in R$.

Example: $f(x + y) = x + y$, $x$ and $y$ unary numbers.

\begin{align*}
\text{start with:} & \quad 111 + 1111 \\
\uparrow & \\
\text{end with:} & \quad 1111111 \\
\uparrow &
\end{align*}
Example: Copy a String, \( f(w) = w0w \),
\( w \in \Sigma^* \), \( \Sigma = \{a, b, c\} \)

Denoted by \( C \)

- start with: \( \text{abac} \)
  \[ \uparrow \]

- end with: \( \text{abac0abac} \)
  \[ \uparrow \]

Algorithm:

- Write a 0 at end of string
- For each symbol in string
  - make a copy of the symbol
Example: Shift the string that is to the left of the tape head to the right, denoted by $S_R$ (shift right).

Below, “ba” is to the left of the tape head, so shift “ba” to the right.

start with: aaBbabc

↑

end with: aaBBbaca

↑
Algorithm:

- remember symbol to the right and erase it
- for each symbol to the left do
  - shift the symbol one cell to the right
- replace first symbol erased
- move tape head to appropriate position
Example: Shift the string that is to the right of tape head to the left, denote by $S_L$ (shift left)

\[
\begin{align*}
\text{start with:} & \quad \text{babcaBba} \\
& \quad \uparrow \\
\text{end with:} & \quad \text{bacaBBba} \\
& \quad \uparrow \\
\end{align*}
\]

(similar to $S_R$)
Example: Add unary numbers
This time use shift.

Example: Multiply two unary numbers, $f(x\ast y) = x\ast y$, $x$ and $y$ unary numbers. Assume $x, y > 0$.

start with: $\text{1111} \ast \text{11}$

end with: $\text{11111111}$