Section: Finite Automata

Deterministic Finite Acceptor (or Automata)

A DFA = (Q, Σ, δ, q₀, F)

where

Q is finite set of states
Σ is tape (input) alphabet
q₀ is initial state
F ⊆ Q is set of final states.
δ : Q × Σ → Q
Example: DFA that accepts even binary numbers.

Transition Diagram:

\[ M = (Q, \Sigma, \delta, q_0, F) = \]

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>q0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>q1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Example of a move: \( \delta(q_0, 1) = \)
Algorithm for DFA:

Start in start state with input on tape
q = current state
s = current symbol on tape
while (s != blank) do
    q = δ(q,s)
    s = next symbol to the right on tape
if q ∈ F then accept

Example of a trace: 11010
Pictorial Example of a trace for 100:

1) \[ \begin{array}{ccc}
1 & 0 & 0 \\
\end{array} \]

\[ \begin{array}{c}
q0 \\
q1 \\
\end{array} \]

2) \[ \begin{array}{ccc}
1 & 0 & 0 \\
\end{array} \]

\[ \begin{array}{c}
q0 \\
q1 \\
\end{array} \]

3) \[ \begin{array}{ccc}
1 & 0 & 0 \\
\end{array} \]

\[ \begin{array}{c}
q0 \\
q1 \\
\end{array} \]

4) \[ \begin{array}{ccc}
1 & 0 & 0 \\
\end{array} \]

\[ \begin{array}{c}
q0 \\
q1 \\
\end{array} \]
Definition:

\[ \delta^*(q, \lambda) = q \]
\[ \delta^*(q, wa) = \delta(\delta^*(q, w), a) \]

Definition The language accepted by a DFA \( M = (Q, \Sigma, \delta, q_0, F) \) is set of all strings on \( \Sigma \) accepted by \( M \). Formally,

\[ L(M) = \{ w \in \Sigma^* \mid \delta^*(q_0, w) \in F \} \]
Trap State

Example: \( L(M) = \{ b^n a \mid n > 0 \} \)
Example:

\[ L = \{ w \in \Sigma^* \mid w \text{ has an even number of } a\text{’s and an even number of } b\text{’s} \} \]
Example: DFA that accepts even binary numbers that have an even number of 1’s.
Definition A language is regular iff there exists DFA $M$ s.t. $L = L(M)$. 
Chapter 2.2

Nondeterministic Finite Automata (or Accepter)

Definition

An NFA = (Q, Σ, δ, q₀, F)

where

Q is finite set of states
Σ is tape (input) alphabet
q₀ is initial state
F ⊆ Q is set of final states.
δ: Q × (Σ ∪ {λ}) → 2^Q
Example

\[
\delta(q_0, a) = L =
\]
Example

$L = \{(ab)^n \mid n > 0\} \cup \{a^n b \mid n > 0\}$
Definition: $q_j \in \delta^*(q_i, w)$ if and only if there is a walk from $q_i$ to $q_j$ labeled $w$.

Example From previous example:

$\delta^*(q_0, ab) =$

$\delta^*(q_0, aba) =$

Definition: For an NFA $M$, 
$L(M) = \{ w \in \Sigma^* \mid \delta^*(q_0, w) \cap F \neq \emptyset \}$
2.3 NFA vs. DFA: Which is more powerful?

Example:
Theorem Given an NFA 
\( M_N = (Q_N, \Sigma, \delta_N, q_0, F_N) \), then there exists a DFA \( M_D = (Q_D, \Sigma, \delta_D, q_0, F_D) \) such that \( L(M_N) = L(M_D) \).

Proof:

We need to define \( M_D \) based on \( M_N \).

\( Q_D = \)

\( F_D = \)

\( \delta_D : \)
Algorithm to construct $M_D$

1. start state is $\{q_0\} \cup \text{closure}(q_0)$

2. While can add an edge
   (a) Choose a state $A = \{q_i, q_j, \ldots q_k\}$ with missing edge for $a \in \Sigma$
   (b) Compute $B = \delta^*(q_i, a) \cup \delta^*(q_j, a) \cup \ldots \cup \delta^*(q_k, a)$
   (c) Add state $B$ if it doesn’t exist
   (d) add edge from $A$ to $B$ with label $a$

3. Identify final states

4. if $\lambda \in L(M_N)$ then make the start state final.
Example:
Minimizing Number of states in DFA

Why?

Algorithm

- Identify states that are indistinguishable

These states form a new state

Definition Two states $p$ and $q$ are indistinguishable if for all $w \in \Sigma^*$

\[
\begin{align*}
\delta^*(q, w) \in F & \Rightarrow \delta^*(p, w) \in F \\
\delta^*(p, w) \not\in F & \Rightarrow \delta^*(q, w) \not\in F
\end{align*}
\]

Definition Two states $p$ and $q$ are distinguishable if $\exists w \in \Sigma^*$ s.t.

\[
\begin{align*}
\delta^*(q, w) \in F & \Rightarrow \delta^*(p, w) \not\in F \text{ OR } \\
\delta^*(q, w) \not\in F & \Rightarrow \delta^*(p, w) \in F
\end{align*}
\]
Example:
Example:
Properties and Proving - Problem 1

Consider the property

Replace_one_a_with_b or R1awb for short. If L is a regular, prove R1awb(L) is regular.

The property R1awb applied to a language L replaces one a in each string with a b. If a string does not have an a, then the string is not in R1awb(L).
Properties and Proving - Problem 2

Consider the property

Truncate_all_preceeding_b’s or TruncPreb for short. If L is a regular, prove TruncPreb(L) is regular.

The property TruncPreb applied to a language L removes all preceeding b’s in each string. If a string does not have an preceeding b, then the string is the same in TruncPreb(L).