Proof of Problem 1. Consider the property $R_{l_{awb}}$. If $L$ is regular, prove $R_{l_{awb}}(L)$ is regular.

**Proof**

Assume $L$ is regular

Exist a DFA $M$ s.t. $L = L(M)$.

$M = (Q, \Sigma, \delta, q_0, F)$

Construct an NFA $\hat{M}$ from $M$ s.t.

$L(\hat{M}) = R_{l_{awb}}(L)$

To construct $\hat{M}$, make a copy of $M$ called $M' = (Q', \Sigma, \delta', q'_0, F')$ where $M'$ is an exact copy with everything primed.

Idea:

Now describe the construction/changes.
\[ M = (Q, \Sigma, \delta, q_0, F) \]

\[ \hat{Q} = Q \cup Q' \quad \text{(states in} \hat{Q} \text{ are states from} M + M') \]

\[ \hat{F} = F' \quad \text{(final states in} \hat{Q} \text{ are final states from} M') \]

\[ \hat{\delta} = \delta \cup \delta' \cup \delta(q, b) = p' \text{ for every } a \text{ arc} \]

\[ \delta(q, a) = p \text{ where } q, p \in Q + Q' \]

\[ \text{(for every} a \text{ arc in} M, \text{ add a} b \text{ arc to the corresponding destination in} M') \]

Let \( w = uav \) Show that is \( w \in L, \text{ then} \]
\[ w' = ubv \in RLawb(L) \]

Suppose \( w \in L \)
\[ \delta^*(q_0, uav) = p \in F \]
\[ \delta^*(q_0, u) = r \in Q, \; \delta(r, a) = s \in Q, \; \delta^*(s, v) = p \in F \]

Thus
\[ \delta^*(q_0, u) = r \in Q, \; \delta^*(r, b) = s' \in Q', \; \delta^*(s', v) = p' \in F' \]

So \( w' = ubv \in F' \), thus \( w' = ubv \in RLawb(L) \)

Suppose \( w \notin L \)
\[ \delta^*(q_0, uav) = p \notin F \]
\[ \delta^*(q_0, u) = r \in Q, \; \delta(r, a) = s \in Q, \; \delta^*(s, v) = p \notin F \]

Thus
\[ \delta^*(q_0, u) = r \in Q, \; \delta^*(r, b) = s' \in Q', \; \delta^*(s', v) = p' \notin F \]

So \( w' = ubv \notin F' \), thus \( w' = ubv \notin RLawb(L) \)