Due Date: February 3, 2015

Problem 1: Describe a 3-tape deterministic TM $M$ that given two natural numbers $x$ and $y$, written on tape-1 in binary, writes $x + y$ on tape-3 in the binary form. You need to describe the set of states, alphabet, initial state, and transition function of $M$. You can assume that the input is of the form $\triangleright x+y$, where the $+$ symbol separates the two numbers. For convenience, you can assume that the least significant bit (resp. most significant bit) is the leftmost (resp. rightmost) bit. For example, if $x = 6$ and $y = 11$, then the input tape contains $\triangleright 011 + 1101 \downarrow$. When the TM stops, the output tape should contain $\triangleright 10001$.

Problem 2: Suppose that we have a TM with an infinite two-dimensional tape (blackboard). There are now moves of the form ↑ and ↓, along with ← and →. The input is written initially to the right of the head position.

(a) Give a detailed definition of the transition function of such a machine. What is a configuration?

(b) Show that such a 2D TM can be simulated by a 3-tape TM with a quadratic loss of efficiency. (Hint: Extend the proof of Theorem 2.1 from the Papadimitriou book.)

Problem 3: Show that the language $L = \{ x \in \{0, 1\}^* \mid x \text{ contains at least two } 0s \text{ but not two consecutive } 0s \}$ can be accepted by a TM with just read-only tape.

Problem 4: Show that the following problems involving TM are not recursive:

(a) Given a TM $M$, is there a string on which $M$ halts?

(b) Given a TM $M$, does it ever write a symbol $\sigma$?

(c) Given a TM $M$, is $L(M)$ empty? ($L(M)$ denotes here the language accepted by $M$, not decided by it.)

Which of these languages are recursively enumerable?

Problem 5: (a) Show that if $L$ is recursively enumerable, then there is a TM $M$ that enumerates $L$ without repeating an element of $L$.

(b) Show that $L$ is recursive if and only if there is a TM $M$ that enumerates the strings of $L$ in length increasing fashion.