Due Date: February 17, 2015

Problem 1: Are the following languages r.e.? Prove your answer.

- **EQ** = \{⟨M⟩; ⟨M’⟩ | L(M) = L(M’)}.
- **NR** = \{⟨M⟩ | L(M) is not recursive\}.

Problem 2: Show that there is an algorithm that given a string x and the encoding of a TM M accepting the language \{(x,y) | (x,y) ∈ R\}, where R is a relation, constructs the encoding of a TM Mx accepting the language \{y | (x,y) ∈ R\}.

Problem 3: Suppose f is a recursive function mapping the encodings of TM to the encodings of TM. Show that there is a TM M such that for all x, M(x) = f(M)(x). That is, M and f(M) are equivalent, i.e., M is a fixpoint of f. (Hint: Let g be the recursive function that maps any TM M to another TM, which on an input x computes M(M)(x)—if M(M) is not a proper description of TM, then assume that M(M)(x) = ?. Let M_{fg} be a TM that computes f ◦ g (the composition of f and g). Show that g(M_{fg}) is the required fixpoint.)

Problem 4: Define the Kleene star of a language L to be

\[ L^* = \{x_1 \ldots x_k | k \geq 0; x_1, \ldots, x_k \in L\}. \]

Show that NP is closed under Kleene star, i.e., if L ∈ NP then L^* ∈ NP. Is P also closed under Kleene star?

Problem 5: Let f, g : \mathbb{N} → \mathbb{N} be two recursive functions. Show that if both f, g ∈ P, then f ◦ g ∈ P. Does a similar statement hold if f, g ∈ EXP?