Due Date: March 05, 2015

**Problem 1:** Consider the language  

\[ DSAT = \{ \varphi \mid \varphi \text{ has at least two satisfying assignments} \} \].

Show that DSAT is NP-Complete.

**Problem 2:** Let \( \varphi \) be a 3CNF-formula. An \( \neq \)-assignment to the variables of \( \varphi \) is one in which each clause contains two literals with unequal values (i.e., one of them is 1 and the other 0). Note that an \( \neq \)-assignment satisfies \( \varphi \) without all literals of any clause of being 1.

- Show that the negation of an \( \neq \)-assignment is also an \( \neq \)-assignment.
- Let \( \neq \)-SAT be the set of 3CNF-formulae that have an \( \neq \)-assignment. Show that 3SAT is polynomial-time reducible to \( \neq \)-3SAT by replacing each clause \( C_i = (y_1 + y_2 + y_3) \) with two clauses \( (y_1 + y_2 + z_i) \) and \( (z_i + y_3 + b) \), where \( z_i \) is a new variable used for \( C_i \) and \( b \) is a single variable used for all clauses.
- Conclude that \( \neq \)-SAT is NP-complete.

**Problem 3:** Given a graph \( G = (V, E) \), a subset \( A \subseteq V \) is called a dominating set of \( G \) if every node in \( V \setminus A \) is adjacent to a node in \( A \). Let DS be the set of all pairs \( (G, k) \) such that \( G \) contains a dominating set of size at most \( k \). Show that DS is NP-Complete. (Hint: Reduce vertex cover to this problem.)

**Problem 4:** Which of the following languages are regular? If the answer is yes then construct a (deterministic or nondeterministic) finite automaton that accepts the language, and if the answer is no then use the pumping lemma to prove your claim. You can use the closure properties of regular languages to justify your answer.

- \( L = a(abb)^* \cup b \)
- \( L = \{0^m1^n0^n \mid m, n \geq 0\} \)
- \( L = \{\omega \mid \omega \in \{0, 1\}^* \text{ is not a palindrome}\} \)
- \( L = \{\omega \mid \omega = a_1b_1 \cdots a_kb_k, a_1, \ldots, a_k \in A, b_1, \ldots, b_k \in B\} \), where \( A, B \) are regular languages.

**Problem 5:** For a string \( \omega = a_1a_2 \cdots a_n \), let \( \omega^R = a_na_{n-1} \cdots a_2a_1 \). Let \( A \) be a regular language, then show that the language \( A^R = \{\omega^R \mid \omega \in A\} \) is also regular.

**Problem 6:** Let \( \Sigma = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \cdots, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\} \)
That is, $\Sigma$ consists of all size 3 columns of 0 and 1. A string in $\Sigma$ gives three rows of binary strings. Consider each row to be a binary number and let

$$\text{SUM} = \{\omega \mid \omega \in \Sigma^* \text{ the bottom row is the sum of the two top rows}\}.$$ 

For example,

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \in \text{SUM}, \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \notin \text{SUM}$$

Show that SUM is regular. (Hint: It's easier to prove the claim for $\text{SUM}^R$. You can use the claim in the previous problem.)