Due Date: March 31, 2015

Problem 1: Let $G$ be an unweighted, undirected graph. Consider the language

$$L = \{ \langle G, a, b, k \rangle \mid G \text{ has a simple path of length at least } k \text{ from } a \text{ to } b \}$$

Show that $L$ is NP-Complete. You can assume the NP-Completeness of the Hamiltonian path problem.

Problem 2: Let $X$ be a set of objects, and let $\mathcal{R} = \{ R_1, \ldots, R_m \} \subseteq 2^X$, i.e., $R_i \subseteq X$, for every $1 \leq i \leq m$. A subset $\mathcal{C} \subseteq \mathcal{R}$ is called a set cover of $(X, \mathcal{R})$ if $\bigcup_{R \in \mathcal{C}} R = X$, i.e., every object in $X$ appears in at least one of the sets of $\mathcal{C}$. The size of $\mathcal{C}$ is the number of sets in $\mathcal{C}$. Let

$$\text{SetCover} = \{ (X, \mathcal{R}, k) \mid \exists \text{ a set cover of } (X, \mathcal{R}) \text{ of size at most } k. \}$$

Show that SetCover is NP-Complete.

Problem 3: Show that

$$L = \{ \langle G \rangle \mid \text{ at least half the total vertices in } G \text{ form an independent set} \}$$

is NP-Complete. You can assume that the independent set problem is NP-Complete.

Problem 4: Let $\text{MULT} = \{ a \# b \# c \mid a, b, c \text{ are binary integers and } a \times b = c \}$. Show that $\text{MULT} \in L$.

Problem 5: (i) Let $\text{EQ}_{\text{REX,ST}} = \{ \langle R, S \rangle \mid R \text{ and } S \text{ are star-free equivalent regular expressions} \}$. Show that $\text{EQ}_{\text{REX,ST}} \in \text{co-NP}$. What goes wrong if $R$ and $S$ have stars? (Hint: It suffices to prove that the complement of this language is in NP.)

(ii) Let $\text{EQ}_{\text{REX}} = \{ \langle R, S \rangle \mid R \text{ and } S \text{ are equivalent regular expressions} \}$. Show that $\text{EQ}_{\text{REX}} \in \text{PSPACE}$. 