Due Date: April 20, 2015

**Problem 1:** Let $F : \{0, 1\}^n \rightarrow \{0, 1\}$ be a Boolean function. Show that there exists a Boolean circuit $C$ of size $O(2^n/n)$ so that $F(x) = C(x)$ for all $x \in \{0, 1\}^*$. (Hint: Combine some of the common terms in the DNF representation of $F$.)

**Problem 2:** Show that if $\text{NP} \subseteq \text{BPP}$ then $\text{RP} = \text{NP}$. (Hint: Show that if SAT can be solved by a randomized machine, then it can be solved by a randomized machine with no false positives.)

**Problem 3:** Consider the following two languages:

CRITICALSAT: Given a CNF Boolean formula $\phi$, is it true that $\phi \in \text{SAT}$ but deleting any clause makes it satisfiable.

UNIQUESAT: Given a CNF Boolean formula $\phi$, is it true that $\phi$ has a unique satisfying assignment.

Show that both CRITICALSAT, UNIQUESAT are in DP.

**Problem 4:** Show that if 3SAT $\leq_p \overline{3\text{SAT}}$ then $\text{PH} = \text{NP}$.

**Problem 5:** Show that $\text{APSPACE} = \text{EXP}$. 