COMPSCI 323 - Computational Microeconomics

Recitation Week 5: Bidding Languages and Expressive Financial Securities

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1   Bidding languages overview

- Motivation for combinatorial auctions: In what aspects are combinatorial auctions more preferable than sequential auction of multiple items?

- Motivation for bidding languages

- OR language - In what ways can it simplify bidding? What kind of valuation functions are not expressible in this language?

- XOR language - Is there a valuation function that cannot be described using this language? Why?

- How can we use "dummy" items to express substitutability in OR language?

- Computational complexities (interpretation complexities) of the bidding languages (in the size of the bid)

2 Another bidding language for combinatorial auctions

A "bid" in this language consists of a graph, whose edges are the items in the auction, and the sum of numbers of all the adjacent vertices are the value of the bundle. For example, we have Jiali's bid as:
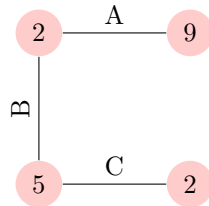
Meaning that his values contain 16 for {A, B}, 7 for {B}, 9 for {B, C}, etc. (all possible combinations of edges in the graph)

Two other bids from Caspar and Hyoung-Yoon are:
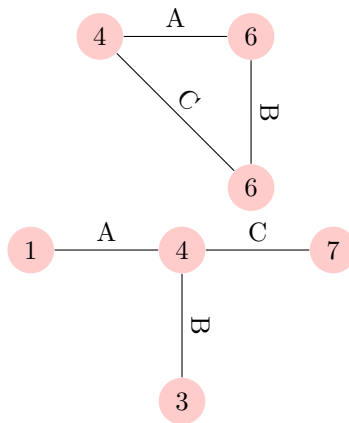
- Can you get the WDP just by looking at the bids?

  Answer: {J-A, C-B, H-C}.

- Why do you think the answer is like this?

  Answer: this biding language indicates strong substitutability among items.

- Can you express complementarities in this bidding language? If so, how?

  Answer: As long as you have negative value on vertex!

3   Miscellaneous questions about financial securities

- Q: Perhaps the simplest financial security is the bond, which pays off k at time t. So at time t, having the bond is always exactly the same as having k. Given that, what are bonds for? Why not just use k whenever we would otherwise use a bond $B(\$k, t)$.

  A: While the two are equivalent at time t, they differ before time t. If you own the actual money, you can use it before t. If you have the bond, you have to wait until time t. In practice, a bond is usually sold when the seller of the bond has a greater need for money now as compared to time t than the seller. For our analyses, bonds are nice because in order to analyze their value, we only have to think about what happens on day t. (This is similar to the reason why we use European-style options rather than American-style options.)

- Q: In lecture, two motivations for trading securities were mentioned. What are they and what are examples of these different motivations?

  A: The two are what is often called *hedging* and *speculating*.

  - Speculating: You might hope to make a profit (in expectation) by buying a security for less than what it is worth (in expectation). (Or by selling it for more than what it is worth.) For example, imagine that you expect a stock to be worth $200 per share in two months. Then you would expect to make a profit if you buy the stock today for a $150 bond. Similarly, you might hope to make a profit by buying call options or selling put options for that stock.

  - Hedging: You might buy securities in order to change the risk profile of your portfolio. For example, imagine that you mostly own stock in a single company. You believe that the stock is worth at least its current price. However, you might worry that in the event that the stock loses its value, you will lose a lot of money. To reduce this downside risk, you could buy put options for that stock to put a lower bound on how much wealth you could end up with.

- Q: Imagine that a particular stock S is currently priced at $200. You believe that the expected price of this stock in the future will also be roughly $200. That is, you believe the price is right in expectation. Most market participants believe that this value will remain quite stable. For some reason, you believe that the chance of a shock of at least $\pm\$50$ is somewhat likely. How might you hope to speculate on that?

  A: The first step is to think about how the market participants' beliefs might be reflected in prices. Note that it cannot be reflected in the price of the share itself. However, it can be expressed in the price of options:

- One might expect that options $C(S, \$230, 7/31/2020)$ are very cheap, because nobody expects that they will be exercised. Hence, you could consider buying this kind of put option.
- Similarly, one might expect that put options like $P(S, \$170, 7/31/2020)$ will be very cheap, because these, too, are not expected to be exercised.

These options are valuable if the price increases/decreases by \$50. (It helps to draw the stock-price/value-graphs for these options!) Hence, you might buy a portfolio consisting of a lot of $C(S, \$230, 7/31/2020)$ and $P(S, \$170, 7/31/2020)$ options. (It helps to draw the graph for this portfolio!)

- Q: Now consider a slightly different setting. You still believe that everybody underestimates the possibility of a $\pm\$50$ change in value. However, now there is also the possibility of a $\pm\$100$ change in value. Everybody (including you) believes this is a serious possibility. Also, you believe that others are much better at predicting the probability of these large shocks as well as the direction of these shocks.

  A: This one is more tricky. Let's look at some natural ideas for securities to buy:

  - As before you do not disagree with other participants about the value of the stock itself. So you shouldn't expect to be able to make a profit by selling/buying S.
  - Consider the call option $C(S, \$230, 7/31/2020)$, which we used effectively in the previous problem. Unfortunately, now the true value of this option depends in great part on the possibility of major shock. By assumption, others are better at assessing this possibility. Hence, you are unable to say for the market price of $C(S, \$230, 7/31/2020)$, whether this price is too low.
  - Similar considerations apply to trading options like $P(S, \$170, 7/31/2020)$, as well as options like $C(S, \$280, 7/31/2020)$ and $P(S, \$120, 7/31/2020)$ (betting only on the big shocks).

  So it turns out that you cannot speculate based on your beliefs using a single security. You cannot actually point to any particular market price for any particular security that you disagree with. (Rather, you disagree with the *relationship* between different prices. You think that the market prices for $C(S, \$230, 7/31/2020)$ and $C(S, \$280, 7/31/2020)$ differ too much.)

  But perhaps you can speculate on your beliefs by buying an entire portfolio? Can you construct a portfolio that pays off if the price of S is at about \$250 or \$150 and otherwise doesn't pay off? If so, you would expect that the other traders will sell you this portfolio at an overall price that is lower than its actual value.

Such a portfolio can be constructed similar to how we created portfolios in class. In particular, we can use the following Butterfly portfolios with a peak at $150 to bet on the $-\$50$ price shock:

$$
\begin{aligned}
C(S,&\$120, 7/31/2020) \\
&- 2C(S, \$150, 7/31/2020) \\
&+ C(S, \$180, 7/31/2020)
\end{aligned} \tag{1}
$$

And the following Butterfly portfolio with a peak at $250 can be used to bet on the $+\$50$ price shock:

$$
\begin{aligned}
C(S,&\$230, 7/31/2020) \\
&- 2C(S, \$250, 7/31/2020) \\
&+ C(S, \$270, 7/31/2020)
\end{aligned} \tag{2}
$$

(Again it helps to look at the graph for these portfolios.)

- Q: By the way, you've seen put-call parity in class. So, can you write the above portfolio using just put options instead of call options?

  A: Sure! In fact, it turns out that you just have to change the Cs into Ps to get the following double Butterfly portfolio:

$$
\begin{aligned}
&+ P(S, \$120, 7/31/2020) \\
&\quad - 2P(S, \$150, 7/31/2020) \\
&\quad + P(S, \$180, 7/31/2020) \\
&+ P(S, \$230, 7/31/2020) \\
&\quad - 2P(S, \$250, 7/31/2020) \\
&\quad + P(S, \$270, 7/31/2020)
\end{aligned} \tag{3}
$$

  Again, it's best to think about this using a graph as in class. With put options it's slightly easier to go through the graph from right to left.

- Q: Does it make sense to use bidding languages in markets for expressive securities?

  A: Yes! The scenario from the previous question is an example in which a trader may want to bid only on a particular set of securities, rather than bidding on different securities individually. And with this combinatorial aspect, the usual problems motivating the creation of bidding languages arise – we certainly don't want traders to have to report for each portfolio much they would be willing to pay for that portfolio.

- Q: When we introduced auctions, we discussed both iterative and single-step mechanisms and how they relate to each other. For example, we have argued that there is some kind of equivalence between the Japanese auction (a.k.a. ascending clock auction) and a Vickrey (sealed-bid second

price) auction. In the both, bidders honestly report their valuation (in the Japanese auction by staying in the room exactly until the price exceeds their valuation) and in both the winner pays the second-highest valuation. We could similarly imagine that a security – e.g., a call option for a particular stock – is auctioned off using either of these auction formats. Are these two mechanisms still equivalent?

A: One very significant issue is the following. Remember that in the Japanese auction you see other bidders leave the room. You also know that these other bidders try just like you to estimate the monetary value of the security at sale. It seems sensible, therefore, that whenever somebody leaves the room, you gain some information about the security at sale. Plausibly the bidder who just left has some private information which persuaded him to leave and you should take this into account when deciding when to leave the room yourself. (Similarly, when nobody leaves the room, there must have been information persuading the other bidders to stay.) An extreme example would be that you have private information which tells you that one particular other bidder underestimates the value of the security by exactly \$1.

The underlying difference here is that the auction for a security is a *common value auction*: the value of the security is the same to everyone. The different bidders only disagree in their *estimates* of that value. In contrast, we have previously considered *private value auctions* in which the bidders disagree about the value of the item on sale even given perfect information. Further, we have imagined in these private auctions, bidders do not update their valuation of an item based on seeing other bidders' valuations. Hence, in the Japanese auction, bidders do not even have to look at when other bidders leave.