COMPSCI 323 - Computational Microeconomics

# Notes for recitation, April 20, 2020: Even More Games

Caspar, Hyoung-Yoon, Jiali, Vince

## 1 Oil price war

A simple game theoretic model can elucidate some basic elements of the ongoing Russia-Saudi oil price war.

Though Trump called the oil price decline "the greatest tax cut," low oil prices can exert a destabilizing influence on the world economy. Particularly, for poor nations that heavily rely on oil production, it is in their vital national interest to maintain stable oil prices.

The virus outbreak has dramatically reduced the global demand for oil, causing a dramatic decline in oil prices. In a situation like this, it may seem that an intuitive solution for some major oil producers like Russia and Saudi Arabia would be to agree on cutting oil production to counterbalance the demand shock. However, this is not what has been realized. Instead, Russia rejected a production cut agreement, instigating the price war.

1. Why would some major producers refuse to lower their production targets despite the losses due to low oil prices? Formulate a game table. What type of game does this remind you of?

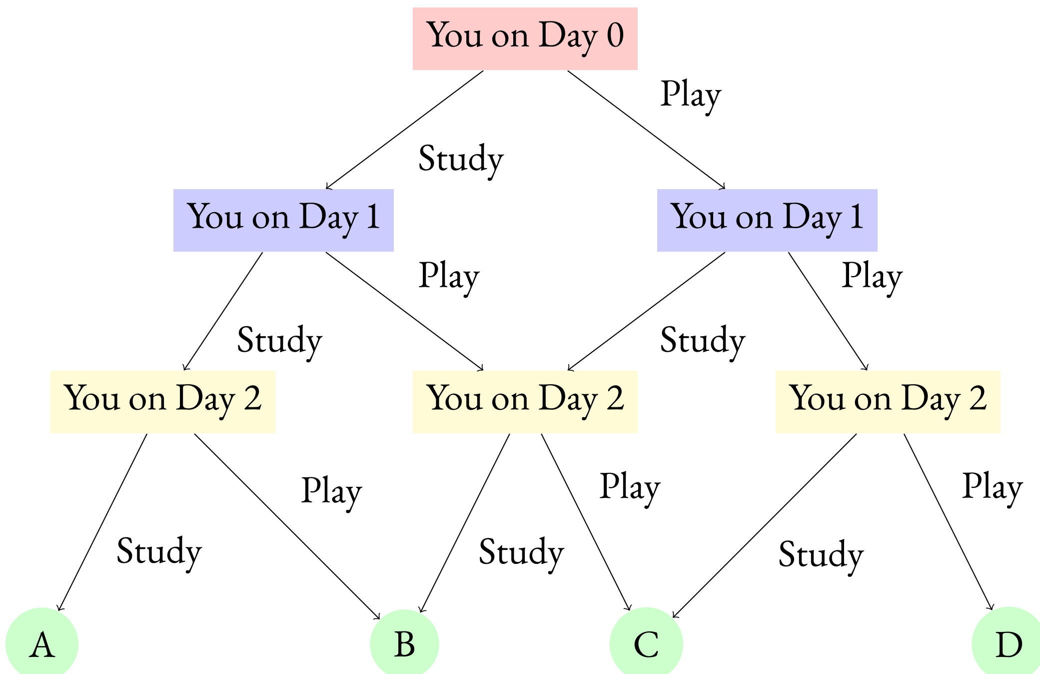
| | | Saudi Arabia | |
|---|---|---|---|
| | | Cut supply | No cut |
| Russia | Cut supply | (-3, -3) | (-5, 0) |
| | No cut | (0, -5) | (-10, -10) |

2. Find a pure-strategy Nash equilibrium, if exists.
   Two pure-strategy NEs: (No cut, Cut supply) and (Cut supply, No cut)

3. Find a mixed-strategy Nash equilibrium, if exists.
   Suppose Saudi Arabia mixed its strategies with probabilities $p$ and $1 - p$. Note that in a mixed-strategy NE, it must be that $U_{RUS}(Cut) = U_{RUS}(Nocut)$. It follows that $(-3)p + (-5)(1-p) = 0 * p + (-10)(1-p)$, which implies that $p = 5/8$. Since it is a symmetric game, the mixed-strategy NE would be that both countries cut supply with probability $5/8$ and do not cut supply with probability $3/8$.

## 2 The Game of Self-Control

Now consider a extensive-form game where player is yourselves, at different time, so they don't have any control over each other. Assume you are "You on Day 0", and there are only three days left for final exams. You are playing this game against "You in the future", they maximize their own payoffs.




Payoffs for each player is the same simple function of the GPA of final letter grade you received, plus 0.5 if you choose to play in thon Day. Thus, $\forall i \in$ {You on Day 0, You on Day 1, You on Day 2}:

$$u_i = 0.5 \times \mathbb{1}_{[\text{Play}]} + \begin{cases} 4, \text{if finally receive A} \\ 3, \text{if finally receive B} \\ 2, \text{if finally receive C} \\ 1, \text{if finally receive D} \end{cases} \tag{1}$$

## 2.(a) What are the subgame perfect equilibria of the game? Why?

**Answer** Study three days and get A. Payoff is 4 and no deviation.

## 2.(b) Now assume that you will get a S/U grade based on new policy. What are the subgame perfect equilibria of the game? Why?

Notice that A grade of S (satisfactory) will be awarded if you earn the equivalent of a letter grade of C- or higher.

Thus, your payoffs are $\forall i \in \{$You on Day 0, You on Day 1, You on Day 2$\}$:

$$u_i = 0.5 \times \mathbb{1}_{[\text{Play}]} + \begin{cases} 3, \text{ if finally receive S (A, B, or C)} \\ 1, \text{ if finally receive U (D)} \end{cases} \tag{2}$$

**Answer** Play two days and study on last day and get C.

## 2.(c) A More Realistic Game

Now consider the same Game with different results (last row). Previous game is additive w.r.t. study/play choices, which is not fun.

Therefore, your final grade does not only depends on how many days you study, but also on how you arrange the days of study. Specifically, we have 3 rules:

1. 3 days of study gives you A-, 2 gives B, B+ or B-, 1 gives C or C+/-, 0 gives D or D+/-.
2. 2-days consecutive study gives you a "+" on top of rule No.1, unless it is 3 days of study.

3. Last-day play gives you a "-" on top of rules No.1 and No.2, unless you have study two consecutive days.

First rule overwrites second when conflict, first and second rules overwrite third when conflict.
In addition, your payoffs are $\forall i \in \{\text{You on Day 0, You on Day 1, You on Day 2}\}$:

$$u_i = 0.5 \times \mathbb{1}_{[\text{Play}]} + GPA \tag{3}$$

Check the GPA of final letter grade.

### (i) What are the payoffs

**Answer** They are 3.7, 3.3; 3.0, 1.7; 3.3, 1.7; 2.0, 1.0

**(ii) Along which path you can achieve your full potential/highest payoff/highest "social welfare"? Is that plan feasible? Why?**

Note that highest "social welfare" may be the sum over 3 player's payoffs.

**Answer** A-: 3.7

**(iii) What are the subgame perfect equilibria?**

**Answer** Study on last two days only.

**(iv) Will you study in the first day? Why?**

**Answer** No.