COMPSCI 323 - Computational Microeconomics

Notes for recitation, April 6: Strategic Dominance

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1 Traveler's Dilemma

Traveler's Dilemma is a non-zero-sum game in which each player proposes a payoff (e.g., between $2 and $100). The lower of the two proposals wins; the lowball player receives the lowball payoff plus a small bonus (e.g., $2), and the highball player receives the same lowball payoff, minus a small penalty (e.g., $2).

1.(a)   Let's play this game

Type your value (between $2 and $100) in chat room and wait until everyone finish typing, we enter at the same time.

1.(b)   What is the Nash equilibrium solution for the game we just played? How to prove it?

Answer $2.

Answer   Show by iterated strict dominance: $99 strictly dominates $100, $98 dominates $99, ..., $2 dominates $3. Or we can show by finitely-iterated prisoner's dilemma.

1.(c)   Did you play Nash equilibrium? Why?

Answer   You don't need to play Nash solution, as sometimes (half of the time) your opponents are naive players. In this game naive play outperform the Nash equilibrium.

1.(d)   Let's play again if the value is between $20 and $100 and bonus/penalty is $20

2   Path independence of strict dominance

Recall from lecture and from the previous section:

- A game consists of two finite sets A_1, A_2 of strategies (or actions), one for each player, and two utility functions u_1, u_2, again one for each player.

- We say that one strategy s_i *strictly dominates* another strategy s_i' if for any opponent strategy $s_{-i} \in A_{-i}$ it is

$$u_i(s_i, s_{-i}) > u_i(s_i', s_{-i}).$$

- Arguably, strictly dominated strategies should not ever be played. We could go further and repeatedly remove strictly dominated strategies.

Question: Let's say we iteratively remove strictly dominated strategies until all strategies that are left are not strictly dominated. At some points in that process we may have the choice between removing different strategies because multiple strategies may be strictly dominated. Do those choices matter for the final sets of strategies that remain?

We will here show that the answer is no, i.e., that iterated strict dominance is path-independent. Remember from lecture that iterated *weak* dominance *is* path-dependent.

We prove this in small manageable steps.

Lemma 1. *Take a game in which some strategy s_i' is strictly dominated. Now let's say that we remove some strictly dominated strategy other than s_i'. Then in the new game s_i' is still strictly dominated.*

Proof. Let s_i be the strategy that strictly dominates s_i'. We distinguish two cases:

Case 1: The strategy removed is s_i. Then there must be $\hat{s}_i$ that strictly dominates s_i. Then it is for all s_{-i}

$$u_i(\hat{s}_i, s_{-i}) > u_i(s_i, s_{-i}) > u_i(s_i', s_{-i}).$$

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Both inequalities are due to the definition of strict dominance. We conclude that $\hat{s}_i$ must strictly dominate s_i'.

Case 2: The strategy removed is one other than s_i or s_i'. Since the set of strategies of the new game is a subset of the strategies of the old game it is still for each strategy s_{-i} in the new game

$$u_i(s_i, s_{-i}) > u_i(s_i', s_{-i}),$$

i.e., s_i' is still strictly dominated by s_i.

$\square$

Lemma 2. *Take a game in which some strategy s_i' is strictly dominated. Now let's say that we iteratively remove any number of strictly dominated strategies other than s_i'. Then in the new game s_i' is still strictly dominated.*

This follows inductively from Lemma 1.

Theorem 3. *Take any game. Iteratively remove strictly dominated strategies until no strictly dominated strategies are left. Regardless of which strategy you remove when multiple strategies are strictly dominated, the final result of the iterative process will be the same.*

Proof. Let's say that two different people execute the iterative process until the end and obtain two results. The first person gets $(\hat{A}_1 \subseteq A_1, \hat{A}_2 \subseteq A_2)$ and the second obtains $(\tilde{A}_1 \subseteq A_1, \tilde{A}_2 \subseteq A_2)$. We need to show that $(\hat{A}_1, \hat{A}_2) = (\tilde{A}_1, \tilde{A}_2)$. We do this by showing that if a strategy s_i is missing in $\hat{A}_i$, it must also be missing in $\tilde{A}_i$. (Technically, we also have to show that if a strategy is missing in $\tilde{A}_i$ it must also be missing in $\hat{A}_i$. But this can be shown in exactly the same way. So it's enough to do one of the proofs.)

So let
$$\hat{s}^1, \hat{s}^2, ..., \hat{s}_k$$
be the sequence of strategies that are removed to obtain $(\hat{A}_1, \hat{A}_2)$. We can show inductively that each of these must also be missing from $\tilde{A}_i$:

- $\hat{s}^1$ is already strictly dominated in the beginning (i.e., in (A_1, A_2, u_1, u_2)). Hence, it cannot be present in $\tilde{A}_1, \tilde{A}_2$. After all, if it was present in either, then by Lemma 2 it would still be strictly dominated. But we have assumed that in $(\tilde{A}_1, \tilde{A}_2)$ no strictly dominated strategies are left.

  Further, by Lemma 2, we could have obtained $(\tilde{A}_1, \tilde{A}_2)$ by removing $\hat{s}^1$ in the very beginning, and then proceeding according to the original procedure.

- $\hat{s}^2$ is strictly dominated once $\hat{s}^1$ is removed. As shown in the previous step, $\tilde{A}_1, \tilde{A}_2$ can be constructed from first removing $\hat{s}^1$ and then iteratively removing strictly dominated strategies. Hence, by the same argument from Lemma 2 as before, $\hat{s}^2$ cannot be in $\tilde{A}_1, \tilde{A}_2$.

  Further, we can again conclude that $(\tilde{A}_1, \tilde{A}_2)$ can be obtained by first removing $\hat{s}^1$ and then $\hat{s}^2$.

- And so forth.

$\square$