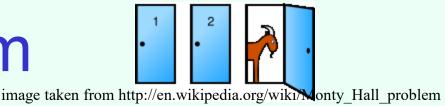
Sleeping Beauty and games of imperfect recall

Instructor: Vincent Conitzer

Monty Hall problem



- Game show participants can choose one of three doors
- One door has a car, two have a goat
 - Assumption: car is preferred to goat
- Participant chooses door, but not opened yet
- At least one of the other doors contains a goat; the (knowing) host will open one such door (flips coin to decide if both have goats)
- Participant is asked whether she wants to switch doors (to the other closed door) – should she?

Imperfect recall

- An Al system can deliberately forget or recall
- Imperfect recall already used in poker-playing
 Al
 - [Waugh et al., 2009; Lanctot et al., 2012; Kroer and Sandholm, 2016]
- But things get weird....







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Enjoy the lazy days of summer and some engaging mathematics in the latest issue of the *Monthly*.

Peter Winkler explores the probabilistic and philosophical conundrums facing Sleeping Beauty and those observing her as she is awakened once or twice during her slumber. Arseniy Akopyan and Vladislav Vysotsky study the relation between the length of a curve that passes through a fixed number of points on

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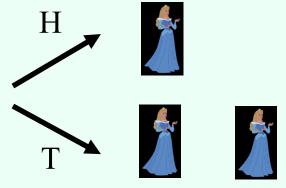
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Sleeping Beauty problem

- There is a participant in a study (call her Sleeping Beauty)
- On Sunday, she is given drugs to fall asleep
- A coin is tossed (H or T)
- If H, she is awoken on Monday, then made to sleep again
- If T, she is awoken Monday, made to sleep again, then again awoken on Tuesday
 Sunday Monday Tuesday



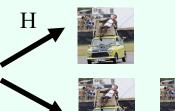
don't do this at home / without IRB approval...

- Due to drugs she cannot remember what day it is or whether she has already been awoken once, but she remembers all the rules
- Imagine you are SB and you've just been awoken. What is your (subjective) probability that the coin came up H?

Modern version

- Low-level autonomy cars with AI that intervenes when driver makes major error
- Does not keep record of such event
- Two types of drivers: Good (1 major error), Bad (2 major errors)
- Upon intervening, what probability should the AI system assign to the driver being good?

Sunday Monday Tuesday



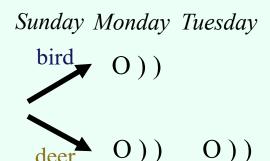




- We place cheap sensors near a highway to monitor (and perhaps warn, with a beep) wildlife
 - Assume sensors don't communicate, don't remember
- Deer will typically set off a sensor twice
- Birds will typically set off a sensor once
- From the perspective of a sensor that has just been set off, what's the probability it's a bird?

Modern version,

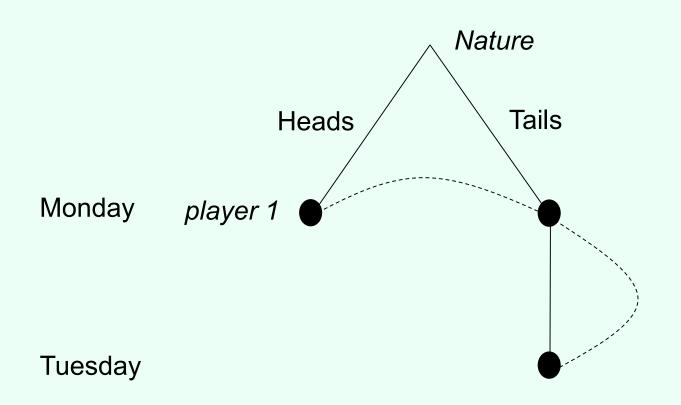
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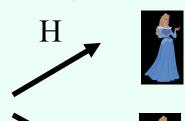
Information structure



Dutch book against Halfer [Hitchcock'04]

- A Dutch book is a set of bets that someone with a particular belief system would each accept, but that in combination lead to a sure loss
- Offer Beauty the following bet whenever she awakens:
 - If the coin landed Heads, Beauty receives 11
 - If it landed Tails, Beauty pays 10
- Argument: Halfer will accept, Thirder won't
- Also offer Beauty on Sunday:
 - If the coin lands Heads, Beauty will pay 12
 - If the coin lands Tails, Beauty will receive 13
- Argument: everyone will accept this one
- If it's Heads, Halfer Beauty will get -12 + 11 = -1
- If it's Tails, Halfer Beauty will get 13 10 10 = -7
- Guaranteed loss!

Sunday Monday Tuesday

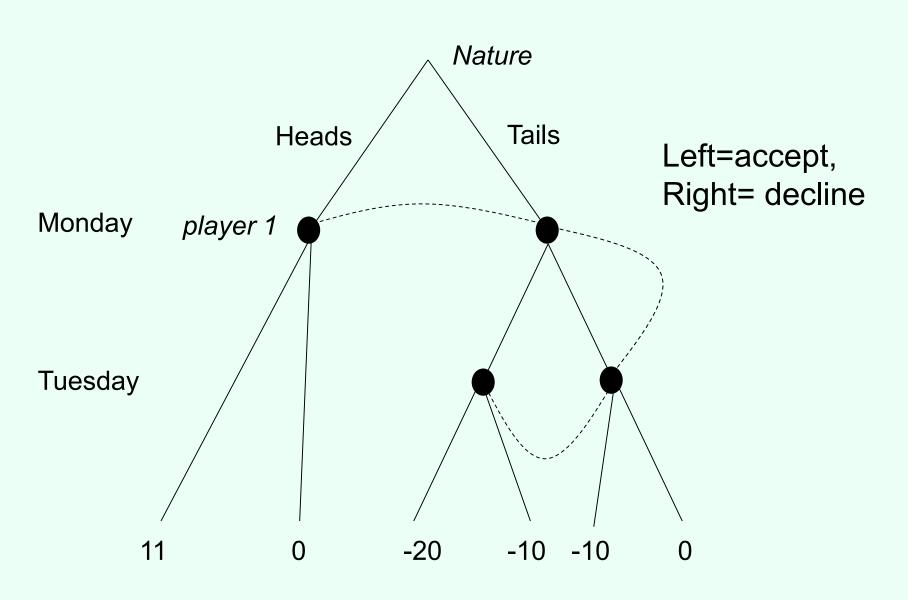








The betting game (ignoring the Sunday bet)

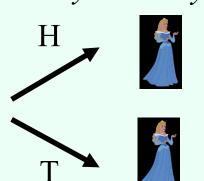


Evidential decision theory

- Idea: when considering how to make a decision, should consider what it would tell you about the world if you made that decision
- EDT Halfer: "With prob. ½, it's Heads; if I accept, I will end up with 11. With prob. ½, it's Tails; if I accept, then I expect to accept the other day as well and end up with -20. I shouldn't accept."
- As opposed to more traditional causal decision theory (CDT)
- CDT Halfer: "With prob. ½, it's Heads; if I accept, it will pay off 11. With prob. ½, it's Tails; if I accept, it will pay off -10. Whatever I do on the other day I can't affect right now. I should accept."
- EDT Thirder can also be Dutch booked
- CDT Thirder and EDT Halfer cannot
 - [Draper & Pust'08, Briggs'10]
- EDTers arguably can in more general setting

[Conitzer'15]

Sunday Monday Tuesday







Dutch book against EDT

[Conitzer 2015]

 Modified version of Sleeping Beauty where she wakes up in rooms of various colors

	WG $(1/4)$	WO $(1/4)$	BO $(1/4)$	BG $(1/4)$
Monday	white	white	black	black
Tuesday	grey	black	white	grey

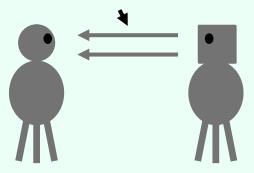
Fig. 3 Sequences of coin tosses and corresponding room colors, as well as their probabilities, in the WBG Sleeping Beauty variant.

	WG $(1/4)$	WO $(1/4)$	BO (1/4)	BG $(1/4)$
Sunday	bet 1: 22	bet 1: -20	bet 1: -20	bet 1: 22
Monday	bet 2: -24	bet 2: 9	bet 2: 9	bet 2: -24
Tuesday	no bet	bet 2: 9	bet 2: 9	no bet
total gain from accepting all bets	-2	-2	-2	-2

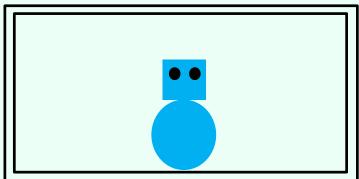
Fig. 4 The table shows which bet is offered when, as well as the net gain from accepting the bet in the corresponding possible world, for the Dutch book presented in this paper.

Philosophy of "being present" somewhere, sometime

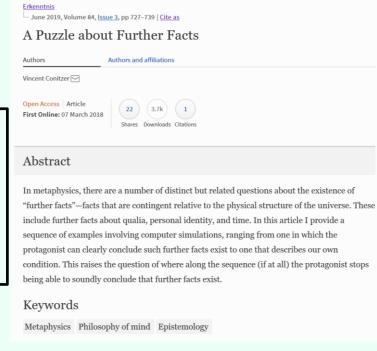
simulated light (no direct correspondence to light in our world)



1: world with creatures simulated on a computer



2: displayed perspective of one of the creatures



See also: [Hare 2007-2010, Valberg 2007, Hellie 2013, Merlo 2016, ...]

- To get from 1 to 2, need additional code to:
 - A. determine in which real-world colors to display perception
 - B. which agent's perspective to display
- Is 2 more like our own experience than 1? If so, are there further facts about presence, perhaps beyond physics as we currently understand it?