Dynamic Programming

Suppose that you are given an nxn checkerboard with numbers in each position, as well as a checker that you want to move across it. You can start at any position on the bottom row, and your goal is to get to any position on the top row. Your checker can only move one position up, up-left, or up-right at a time.

If you make d dollars by landing on a spot with the number d, how do you get across the board with the most money?
Given an nxn grid of nodes, how many paths are there from (1,1) to (n,n) with only up or right moves allowed?
Hashing

<table>
<thead>
<tr>
<th>Action</th>
<th>Key</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Add</td>
<td>31</td>
<td>John</td>
</tr>
<tr>
<td>Add</td>
<td>18</td>
<td>Sam</td>
</tr>
<tr>
<td>Add</td>
<td>2</td>
<td>Sue</td>
</tr>
<tr>
<td>Add</td>
<td>21</td>
<td>Amy</td>
</tr>
<tr>
<td>Add</td>
<td>5</td>
<td>Bill</td>
</tr>
<tr>
<td>Add</td>
<td>13</td>
<td>Dan</td>
</tr>
<tr>
<td>Add</td>
<td>44</td>
<td>Jim</td>
</tr>
</tbody>
</table>

Delete 21

Find 44
Add 31 - John
Add 18 - Sam
Add 2 - Sue
Add 21 - Amy
Add 5 - Bill
Add 13 - Dan
Add 44 - Jim

Delete 21

Find 44

\[ h'(k) = k \mod m \]

\( m = 13 \)

Quadratic

\[ h(k,i) = (h'(k) + c_1 i + c_2 i^2) \mod m \]

\( c_1 = 0, \quad c_2 = 1 \)

\[ 44 + 9 = 53 \]

\[ 53 \mod 13 \]

\( \equiv 1 \mod 13 \)
Add 31 - John
Add 18 - Sam
Add 2 - Sue
Add 21 - Amy
Add 5 - Bill
Add 13 - Dan
Add 44 - Jim

Delete 21
Find 44

\[ h'(k) = k \mod m \]
\[ m = 13 \]

Double Hashing

\[ h(k_i) = (h_1(k) + i \cdot h_2(k)) \mod m \]
\[ h_2(k) = 1 + (k \mod (m-1)) \]
Recurrence Relations - Substitution Method

\[ T(n) = 4T(n/2) + n \]

\[ 0(n^2) \]

\[ \log_2 n \]

\[ \sum_{i=0}^{\log n} (2^i n) \]

\[ = n \sum (2^i) \]

\[ = n (2^{\log n+1} - 1) \]

\[ = 2n 2^{\log n} - n \]

\[ = 2nn \log^2 = 2n^2 \]
\[ T(n) = 10T(\lg n) + n \]
Recurrence Relations - Recursion Tree

\[ T(n) = T\left(2\frac{n}{3}\right) + T\left(\frac{n}{3}\right) + n \]