21 - Maximum Flow

Capacity - How much can go through each edge
Flow - How much is going through each edge
Directed!

 Capacities: 

 Flow value: total leaving source or going to sink.

 Residual:

 $r(s, a)$

 $r(a, s)$
1) \( f(u,v) \leq c(u,v) \)
2) \( f(u,v) = -f(v,u) \)
3) \( \sum_{v \in V} f(u,v) = 0 \) unless \( u=s \) or \( u=t \)
Ford-Fulkerson

Initialize flow graph to zeros

While a residual path $p$ from $s$ to $t$ exists $\leq O(V+E)$

$$\min f = \min r(u,v) : (u,v) \text{ is in } p \leq O(V)$$

for each edge $(u,v)$ in $p \leq O(V)$

- $r(u,v) = r(u,v) - \min f$
- $r(v,u) = r(v,u) + \min f$

$$f(u,v) = f(u,v) + \min f$$
$$f(v,u) = -f(u,v)$$

$\sum \leq O(1)$

Total: $O((V+E) \cdot |f^*|)$

Residual Graph $(r(u,v))$

Flow Graph $(f(u,v))$
Problem:

Solution: Edmonds-Karp

Path through residual is called an augmenting path.

\( O((V+E) \cdot E \cdot V) = O((V+E)EV) = O(E^2V) \) path.

push-relabel: \( O(EV^2) \)
relabel-to-front: \( O(V^3) \)
Multiple Source, Multiple Sink
Maximum Bipartite Matching

Matching: Choice of a subset of edges such that no vertex has more than one edge incident upon it.

Maximum matching: matching of largest size.