25 - Tractable vs. Intractable

Problem: "Addition"

<table>
<thead>
<tr>
<th>Instances</th>
<th>Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;2+2&quot;</td>
<td>79</td>
</tr>
<tr>
<td>&quot;1+2&quot;</td>
<td>152</td>
</tr>
<tr>
<td>&quot;38+50&quot;</td>
<td>1</td>
</tr>
<tr>
<td>...</td>
<td>-37</td>
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Problem: function from set of instances $I$ to set of solutions $S$

$I \rightarrow S$
The Halting Problem

\texttt{halt(Algorithm \ a, \ Input \ \ i)}

If alg a runs on input \ i, will it halt?

\texttt{trouble(String \ s) \& \&}
\texttt{if \ halt(s,s) == false}
\texttt{return \ true}
\texttt{else}
\texttt{loop \ forever}
\texttt{)}
\texttt{trouble("trouble");}
Decision Problems

Function from class of instances $I \rightarrow \{0, 1\}$

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<tr>
<td>2+2</td>
<td>0</td>
</tr>
<tr>
<td>10+2</td>
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Optimization problems have been covered more

Shortest Path:

"Is there a path between u and v with length $\leq k$?"
What is \( P \)?

\[
polynomial = O(n^k) \text{ where } k > 0
\]

Polynomial-time solvable: There is an algorithm that produces a solution for any input of size \( n \) in \( O(n^k) \) time

(even if the algorithm is not currently known).

\[
\text{polynomial: } \quad n!, \quad 2^n, \quad n^k
\]
Verifiability

Given an input and a "solution string", can you verify the solution in polynomial time.

NP
NP

P ⊆ NP
Non-determinism

\( \sim 3^n \)

NP

\( \times \times \times \times \)
Does $P = NP$?

$NP = O(2^n)$

$n! \neq NP$

3-SAT: Boolean Formula in 3-CNF, is there a satisfying assignment

$(x_1 \lor x_2 \lor \overline{x_3}) \land (\ldots \lor \ldots \lor \ldots) \land (\ldots \land \ldots) \land$