26 - NP-Completeness

3SAT: Given a boolean formula in CNF, is there a satisfying assignment?

\[ \phi = (x_1 \lor \overline{x}_2 \lor \overline{x}_3) \land (\overline{x}_1 \lor x_2 \lor x_3) \land (x_1 \lor x_2 \lor x_3) \]

\[ \overline{3SAT} : \text{Given formula, is it not satisfiable?} \]

co-NP
Reductions

Hard Problem → Transformation → Unknown Problem

'Yes' → 'Yes'

'No' → 'No'

NP-complete

3SAT

in poly-time

CLIQUE
NP-Hard: Class of all problems at least as hard as all problems in NP.
NP-Complete

Is there a “hardest” problem in NP? Yes.

3SAT

NP-Complete if:

1) Problem is NP-hard and
2) Problem is NP

Proofs:

1) Show it’s in NP

2) Show it’s at least as hard as an NP-complete problem.
Other NPC problems

CLIQUE → Does graph G have a clique of size k?

Is CLIQUE ∈ NP? Yes
\[ \phi = (x_1 \lor \overline{x}_2 \lor \overline{x}_3) \land (\overline{x}_1 \lor x_2 \lor x_3) \land (x_1 \lor x_2 \lor x_3) \]

Clique of size \( k = \# \) of clauses
VERTEX-COVER — Does graph $G$ have a vertex cover of size $k$?