Complexity Qual Spring 2016

Problem 1: Regular Languages:

Write regular expressions for the following languages:

Problem 1(a): Give a regular expression for the set of all strings of 0's and 1's not containing 101 as a substring.

Problem 1(b): Give a regular expression for the set of all strings of 0's and 1's whose number of 0's is divisible by 5 and whose number of 1's is even.

Problem 2: Pushdown Machines:

Let A = $\{a^i b^j c^k \mid C(i = j \text{ or } i = k) \text{ where } i, j, k \geq 0\}$.
Describe a pushdown automaton that recognizes A (an English description will suffice).

Problem 3: Countability:

Problem 3(a): Show that the set of all functions from the natural numbers to $\{0, 1\}$ is uncountable.

Problem 3(b): Now show that the following subset of the above functions is countable: in every function, all odd numbers are mapped to 0 and all even numbers to 1, except exactly one odd number that is mapped to 1 and exactly one even number than is mapped to 0.

Problem 4: Recursive and Enumerable Problems:

Problem 4(a): Show that the following problems involving a Turing Machine(TM) are not recursive:

(i) Given a TM M, does it ever write a particular symbol a ?
(ii) Given a TM M, is L(M) empty?
(L(M) denotes here the language accepted by M.)

Problem 4(b): Which of these languages are recursively enumerable:
(i) The set of TM M, that do not ever write a particular symbol a.
(ii) The set of TM M, where L(M) is empty.
Be sure to justify your answers.

Problem 5: NP-Completeness:

The **reachability relation** R of a directed graph (digraph) G=(V, E) is a relation over V x V such that for each pair of vertices u, v in V:

u R v if and only if there is a path from u to v.

A minimum equivalent digraph is a subgraph of a given digraph that has the same reachability relation as the original digraph and as few edges as possible.

Prove that finding the minimum equivalent digraph is an NP complete problem, using one of 3-SAT, vertex cover, clique, independent set, or Hamiltonian cycle to reduce to the minimum equivalent graph problem.

Problem 6: Nondeterministic LOG Space (NLOG):

Let NLOG be the set of languages accepted by nondeterministic O(log n) space Turing Machines. Let DPATH be the problem: given a directed graph G =(V,E) and also given two vertices s, t in V, determine if there is a directed path from s to t in G. Then show that DPATH is a complete problem for NLOG with respect to deterministic log-space reductions**.** Do this in stages:

Problem 6(a): Define what is a deterministic log-space reduction between two languages L and L'.

Problem 6(b): Show DPATH is in NLOG.

Problem 6(c): Show there is a deterministic log-space reduction from each problem in NLOG to a problem in DPATH.